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MTP-AERO-63-27
April 19, 1963
44 p.

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GEORGE C. MARSHALL SPACE FLIGHT CENTER

HUNTSVILLE, ALABAMA

LUNAR FLIGHT STUDY SERIES: VOLUME 6

A STUDY OF GEOMETRICAL AND TERMINAL CHARACTERISTICS OF EARTH-MOON TRANSITS EMBEDDED IN THE EARTH-MOON PLANE

By

B. J. Lisle

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ABSTRACT 20207

This report represents the results of a study of coplanar earth-moon transits. The study was initiated to provide information concerning coplanar geometrical characteristics of earth-moon transits. The geometrical aspects of transit behavior are related to variations in injection conditions.

The model of the earth-moon system used in this investigation is the Jacobian model of the restricted three body problem. All transits considered in this study are restricted to the moon-earth plane (MEP).

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B. J. Lisle

FUTURE PROJECTS BRANCH
AEROBALLISTICS DIVISION

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SUMMARY

This report represents the results of a study of coplanar earth-moon transits. The study was initiated to provide information concerning coplanar geometrical characteristics of earth-moon transits. The geometrical aspects of transit behavior are related to variations in injection conditions.

The model of the earth-moon system used in this investigation is the Jacobian model of the restricted three body problem. All transits considered in this study are restricted to the earth-moon plane (MEP).

SECTION I. INTRODUCTION

The objective of this investigation is to provide empirical relationships between terminal conditions of earth-moon transits while stressing the geometrical aspects of these transits. The analysis of these relationships has basically the purpose of emphasizing useful characteristics of earth-moon transits.

SECTION II. DISCUSSION

A. THE SYSTEM MODEL

The model of the earth-moon system used in this investigation is the Jacobian model of the restricted three body problem. The gravitational fields are those of the earth and moon while the effects of the probe are neglected due to the relative size of the mass. All transits are assumed to move in the field of the two bodies independent of other forces.

The spherical earth and moon in the system are restricted to motion about the mass center of the system (barycenter). Separation of the two bodies is maintained at a constant distance although in actuality the bodies vary about a mean value by ±20,000 km. The assumption of constant separation distance restricts the system to circular motion and constant angular velocity. The mean distance of separation is 385,080 km with an associated angular velocity of .00015 degree per second. One portion of this report is devoted to evaluating the effects of varying the moon-earth distance and angular velocity of the system (also assuming circular motion).

B. CARTESIAN COORDINATE SYSTEMS

The transits generated for this investigation are described as being coplanar transits. The plane of these transits is the plane of earth-moon motion (MEP). Restriction of all motion to the moon-earth plane simplifies the presentation of geometrical characteristics allowing a more concise interpretation of results.

There are four essentially separate cartesian coordinate systems used in the presentation of results and all are restricted to the moon-earth plane (MEP):

1. The Space-Fixed Coordinate System (Figure 1) has the origin at the barycenter of the system. The x_S axis is directed from the barycenter and passes through the center of the moon at the initial time. The y_S axis of the space-fixed system is perpendicular to the x_S axis at the barycenter and is in the plane of motion described by the two bodies (MEP).

2. The Rotating Coordinate System (Figure 1) has the origin at the barycenter. The x_R axis is defined positive in the direction of the moon and passes through the center of the moon at all times. The negative x_R axis passes through the center of the earth at all times. The y_R axis is perpendicular to the x_R axis at the barycenter and rotates with the system as the x_R axis. The x_R and y_R axes both are in the plane of motion of the two bodies (MEP).

3. The Earth-Centered Coordinate System (Figure 2) has the origin at the center of the earth with the x_E axis defined positive in the direction of the center of the moon. The y_E axis is perpendicular to the x_E axis at the earth center and positive in a westerly direction (opposite in direction to the earth's system motion). This system is in the moon-earth plane.

4. The Moon Centered Coordinate System (Figure 2) has the origin at the center of the moon with the x_M axis defined negative in the direction of the earth center. The y_M axis is perpendicular to the x_M axis at the moon center and positive in the direction of the moon's system motion. This coordinate system lies in the moon-earth plane (MEP).

C. TERMINAL PARAMETERS

The geometrical characteristics of earth-moon transits are best described utilizing the cartesian coordinate systems described above. Terminal parameters are best described and associated when presented in polar coordinate form.

In the restricted system five injection parameters are available for variations. These parameters are: velocity (v_E, v_M), altitude (Alt_E, Alt_M), Path Angle (ϑ_E, ϑ_M), longitude (λ_E, λ_M), and two discrete values of azimuth (Az_E, Az_M). All motion being restricted (MEP) these azimuth may have values of 90 degrees and 270 degrees (i.e., co-rotational and counter-rotational to the system motion). This study is restricted to earth to moon transits. Thus, the injection parameters at the earth (subscript E) are available for variation.

The velocities (v_E, v_M) are both defined in the rotating coordinate system. v_M is referenced to some terminal condition of the transit which has been achieved at the moon (i.e., impact, etc.). Altitudes (Alt_E, Alt_M) are measured along the radius vector from the center of the respective body. The bodies of the system are assumed spherical in form, consequently, the altitudes are the distance from the surface of the body to the probe. Path angles (ϑ_E, ϑ_M) are measured with respect to the local radius vector of the reference body. The path angles are restricted to values from 0 degrees to 180 degrees. Longitudes (λ_E, λ_M) are measured counter-clock wise for both bodies (Figure 2). Zero longitude for the earth is defined as the point farthest away from the moon on the x_R axis. Zero longitude for the moon is defined as the point nearest to the earth on the x_R axis. The azimuths (Az_E, Az_M) are both measured clockwise from north.

SECTION III. RESULTS

A. PERPENDICULAR IMPACTS ON THE LUNAR SURFACE

1. Perpendicular Impacts for the Mean Earth-Moon Distance

The transits which terminate at the moon in a perpendicular impact are taken as characterizing the family of transits that are studied. In generating these transits, the injection altitude is maintained at an earth altitude of 185 kilometers and injection is at perigee (earth path angle of 90 degrees). This family of transits is subdivided into those which leave the earth as co-rotational and counter-rotational transits (i.e., earth azimuth of 90 degrees and 270 degrees, respectively). Under these constraints, the perpendicular impact transits are functions of only injection velocity and longitude. Through the proper combination of injection velocity and longitude, the family of transits which terminate in perpendicular impact at various transit times is generated. The geometrical behavior of a selected group of these transits in a space-fixed reference system is presented in Figure 3.

The geometrical characteristics of these transits are elliptical in form for all except the portion of the path near the moon. All co-rotational injection velocities and most counter-rotational velocities considered are less than local escape velocity. Those transits having the shorter transit times (less than 112 hours) reach the moon prior to apogee of their transit. The transits with long transit times (longer than 112 hours) reach the moon after apogee of their transit. This characteristic is achieved by positioning of the transit (injection longitude). As an illustration, the 84 hour transit and the 167 hour transit have essentially the same injection velocity (v_E = 10,858 meters per second) at co-rotational injection. The injection longitudes are 52 and 90 degrees, respectively, and both transits terminate at the moon in a perpendicular impact. This feature is illustrated geometrically in Figure 3. The 84 hour co-rotational transit impacts the lunar surface while on the ascending portion of the transit and the 167 hour transit has passed through apogee and is returning to earth when lunar impact occurs.

In Figure 4 the transit geometry of the four perpendicular impacts are presented in the rotating coordinate system. The large variation in transit time required for reaching the moon is emphasized in this figure. It is also noted (as in Figure 3) that the longer transit times require the transit to go beyond the earth-moon orbital distance. As an illustration, the 350 hour counter-rotational transit remains beyond the earth-moon distance from the 60th hour of transit to the time of impact.

Figure 5 presents the four transits (co-rotational injection) in a modified coordinate system. The system aligns the line of apsides of each transit such that they are coincident. By so aligning the lines of apsides, it is seen clearly that the difference in transit time is caused by the portion of the transit in which the moon is intercepted. The elliptical nature of all the transits is also clearly depicted.

The injection (v_E, λ_E) and impact (v_M, λ_M) conditions for perpendicular impact transits are given in Figures 6 and 7. These terminal conditions are presented versus transit time from earth to moon. The conditions for both co-rotational and counter-rotational transits are presented. In Figure 6 the terminal velocities presented indicate a minimum velocity required for injection and also a minimum arrival velocity associated with the transit time of approximately 112 hours. This minimum velocity produces a transit which terminates at approximately the apogee of the ellipse, as would be produced in absence of the moon. In Figure 7 the injection and arrival longitudes are presented versus earth-moon transit time. The trend in injection longitude is to progress from the side of the earth which is away from the moon to the side closest to the moon with increased transit time. This is the case for both counter and co-rotational injections. On the lunar surface, the arrival location progresses from the front leading side to the back leading side as transit time is increased.

It is noted that for the 132 hour transit, the injection longitude for counter and co-rotational injection coincide. At this injection longitude, a transit on either injection azimuth ($Az_E = 90^o$ or 270^o) will terminate at the moon in the same transit time and at essentially the same location.

2. Perpendicular Impacts from Varied Injection Perigee Altitudes

Figure 8 presents the injection and arrival geometry of three co-rotational transits, each terminating in a perpendicular impact and having a 60 hour transit time from perigee. It is noted that the perigee for two of the transits lies beneath the surface of the earth. The third transit is of the family of transits previously discussed having a perigee at an altitude of 185 kilometers. The perigee altitude of the lower transits lies 300 and 600 kilometers below the highest altitude transit perigee. Each transit has the same transit time from perigee to perpendicular impact. The arrival conditions (Figure 8) indicate the geometry at impact to be similar with longitude varying by only 0.5 degree at the extreme. The arrival velocities of these transits varies by 2.5 meters per second at the extreme.

Figure 9 indicates the variation in perigee velocity required by each of the transits. When a given altitude is selected for injection, 185 kilometers, the transits from varied perigee altitudes require the same injection velocities although the path angles vary considerably as shown in Figure 10. In Figure 10 the variation in path angle (ϑ_E) versus longitude of injection (λ_E) is presented for transits having equal times from perigee to impact. Here again, the path angle is referenced to a 185 kilometer injection altitude. The time variation in transit time is also presented in Figure 10. The transit time from injection to impact will be diminished by the time from perigee (Δt) to injection as a function of injection longitude (λ_E). Should another injection altitude be selected (other than 185 kilometers) the injection requirements would also be changed.

3. Perpendicular Impacts for Varied Earth-Moon Distances

The earth-moon model used in this study is restricted to circular motion about the barycenter. The effects of variations in earth-moon distance are investigated in a restricted manner. The restriction being the circular orbit and corresponding velocity associated with the earth-moon distances. In Figures 11 and 12 the results on terminal conditions of changes in the earth-moon distance are presented. In these figures the terminal conditions (v_E, λ_E, v_M, and λ_M) are given for three earth-moon distances of 365, 385 and 405 thousand kilometers.

The shortest earth-moon distance requires the minimum injection velocity for a given transit time. The minimum velocity for this distance to attain a perpendicular impact being approximately 10,839 meters per second. When the moon is at the maximum distance, a transit having the same flight duration (~103 hours) requires a velocity of 10,859 meters per second. It is noted that the minimum velocity requirement for each earth-moon distance is associated with slightly different transit times, (i.e., maximum distance ~114 hours, minimum distance ~103 hours).

The arrival velocities (Figure 11) exhibit a characteristic which was inferred previously. For transit times less than 98 hours, the shortest earth-moon distance also has the lowest arrival velocity. For transit times larger than 103 hours, the minimum earth-moon distance exhibits the maximum arrival velocity although the injection velocity remains lower than for the other distances. This feature is, at least in part, due to the geometry of the orbit as discussed previously.

Figure 12 gives the injection and arrival locations with respect to the two bodies. The longitudes of injection and arrival are slightly displaced by a variation in earth-moon distance although the general trends remain essentially the same for all earth-moon separation distances.

B. NON-PERPENDICULAR IMPACTS ON THE LUNAR SURFACE

1. Non-Perpendicular Impacts for Constant Injection Velocities

This section of the report is concerned with transits which impact the lunar surface although not perpendicularly. In Figure 13 a series of these transits are presented. The three transits are identical in injection velocity, path angle and altitude. They differ only in injection longitude. The injection geometry (Figure 13) covers an area approximately 3.5 degrees in longitude of the earth. Through this variation, the arrival conditions are formed at the moon (Figure 13, "Impact Geometry"). From Figure 13 a variation of 3.5 degrees at the earth's surface in injection longitude results in more than 280 degrees longitude coverage of the lunar surface. The perpendicular impact location is given to indicate a familiar reference. The perpendicular impact transit approximately bisects the area which can be covered on the lunar surface.

One other point to be noted is the variation in transit time. The reference perpendicular impact has a transit time of 84 hours. The transit which arrives co-rotational to the moon's motion has a transit time of 81.8 hours while the transit arriving counter-rotational has a transit time of 87.7 hours.

In Figure 14 the effects of injection longitude variations on impact locations are presented. This figure consists of lunar longitude versus transit time. Indicated are the longitudes associated with grazing and perpendicular impacts on the lunar surface. The parameter indicated is injection velocity which is constant for all combinations of lunar longitude and transit times lying on the dashed lines. Each of the constant velocity parameter curves is generated by a variation in injection longitude. As an example, choose an injection velocity of 10,853.5 meters per second. If this velocity is associated with a longitude (λ_E) of 56 degrees, a transit terminating in a grazing impact with an azimuth of 90 degrees is achieved on the moon. As the injection longitude at the earth is increased, the arrival longitude varies along the dashed line as indicated on Figure 14. The impact location progresses across the lunar surface, with increasing injection longitude until grazing impact is achieved on the opposite side of the moon (λ_M = ~135 degrees). This grazing impact corresponds to an earth injection longitude of 60 degrees. As the impact progresses across the lunar surface, the impact path angle varies from 90 degrees to 180 degrees (i.e., perpendicular impact) and back to 90 degrees. For the velocity under consideration, the perpendicular impact occurs at a lunar longitude of 285 degrees. If the variation of earth longitude is continued beyond 60 degrees, the resulting transit will not impact on the lunar surface until the injection longitude reaches 73 degrees. At this point, a grazing impact is again achieved with an associated azimuth of 270 degrees. Continued increase of injection longitude produces a second perpendicular impact at a lunar longitude of 263 degrees and also a fourth grazing impact with an associated azimuth of 90 degrees. This "quadratic effect" will be discussed later in relation to free return transits. It is noted that the transits which impact the lunar surface on one side of the perpendicular impact location (Figure 14) reach the moon with a 90 degree azimuth while the transits arriving to the other side of the perpendicular impact have an azimuth of 270 degrees. This is the system used to differentiate between co-rotational and counter-rotational transits at the moon.

2. Non-Perpendicular Impacts for Constant Injection Longitudes

In Figure 15 injection longitude is presented as the parameter for the same variables presented in Figure 14 (i.e., impact longitude versus transit time). Using injection longitude as a parameter does not reveal a "quadratic effect" as injection velocity in the previous figure.

Comparing Figures 14 and 15 with respect to their parameters (v_E, λ_E) it is noted that the variations from perpendicular impact conditions required to achieve grazing impacts is progressively smaller as larger transit times are achieved. In general, the transits requiring long transit times to impact are very sensitive to injection conditions.

The lunar surface coverage possible through injection velocity and longitude variations are given in Figure 16. The maximum coverage occurs for a transit time of approximately 112 hours. For this transit time approximately 285 degrees of the lunar surface may be covered and impact at any point on the surface area which is accessible may be achieved in the 112 hour transit time. In general, if any transit time is allowable, any point on the lunar surface may be achieved by direct impact (see Figures 14 and 15).

3. Non-Perpendicular Impacts for Constant Transit Times

In the previous section the parameters of injection velocity and longitude have been studied separately with regard to their effects on impact longitude and transit time. If Figures 14 and 15 are compared, it is noted that the lines indicating grazing impact are coincident although the dashed lines indicating constant injection velocity and longitude are not coincident. The figures themselves indicate the possibility of achieving grazing impact on both sides of the lunar sphere and perpendicular impact with the same transit time. The selection of the correct combination of injection parameters will produce equal time transits for perpendicular and grazing impact.

C. LUNAR FLY-BY TRANSITS

1. Locus of Periselena for Constant Injection Velocities

In the previous chapter, grazing impacts were shown to be the limiting impact on the surface of the moon. If a variation in the injection parameters (λ_E, v_E) is continued beyond the magnitude resulting in lunar grazing impact and the transit is terminated at a lunar path angle (ϑ_M) of 90 degrees, a locus of points is described by the termination point of each transit. This locus of points is designated the locus of periselena. In the previous chapter it was also noted that grazing impacts occur approximately 270 degrees apart (variation occurs with transit time, Figure 16). This indicates that the locus of periselenum could be generated as termination point of the family of transits comes closer to the moon and as they pass beyond the moon. This is the case, and in Figure 17 transit geometry is presented. If the injection longitudes are compared for the transits terminating on the loci of periselena to those for the grazing impacts in Figure 13, it is noted that the injection longitudes indicate in Figure 17 bracket those presented in Figure 13. Actually, the transits terminating at periselenum, grazing impact, and perpendicular impact are the result of a continuous injection longitude variation. Comparable features are achieved by maintaining injection longitude constant and changing injection velocity.

Figure 18 presents a selected group of loci of periselena. These loci of periselena are associated with the indicated perpendicular impacts and have the same velocity although not the same injection longitude. The most apparent feature of the loci of periselena is their location change as a function of transit time. The locus of periselena moves in a clockwise (i.e., decreasing longitude) direction as transit time increases. This feature allows the complete impact coverage of the lunar surface (i.e., previous section) and in some instances special symmetrical transits are found for lunar fly-by. The symmetrical transits are free return transits and will be discussed in a following portion of this report.

It is noted, however, that all locations of periselenum are not achievable within the injection velocities of this report. The lunar area consisting of the front leading quadrant and a portion of the back leading quadrant are inaccessible with regard to achieving periselenum.

2. Locus of Periselena of Constant Injection Longitudes

The loci of periselenum for constant injection longitudes are those which have the injection longitude of the associated perpendicular impact transit and are generated by variation of injection velocity. Figure 19 presents this information, and it is noted that there is little difference in location of the loci of periselenum in Figures 18 and 19 at the vicinity of the lunar surface.

3. Locus of Periselena for Constant Transit Time

In the discussion of Figures 14 and 15, it was noted that constant transit time could be achieved for any desired impact location on the lunar surface within the accessible region. It was found that the loci of periselena could also be generated for constant transit times although all longitudes of arrival may not be achieved in the vicinity of the moon. As mentioned in the previous paragraph, Figure 20 gives the location of the loci of periselenum for constant transit times. Again, it is noted that the location of the loci of periselenum is little changed from the two previous figures. It is noted that the constant transit times are achieved by variation of injection longitude and velocity simultaneously.

4. The Line of Vertices

The next geometrical feature of coplanar earth-moon transits to be discussed is the line of vertices. The line of vertices is a locus of points defined by the intersection of circumlunar transits. The line of vertices is a line of transit symmetry. In general, it lies approximately 180 degrees in lunar longitude from the location of perpendicular impact and bisects the area between the loci of periselenum. The examples presented in Figures 21 and 22 are generated by the injection conditions associated with the respective perpendicular impacts except for injection longitude. Each pair of transits has a common lunar altitude at periselenum, the lowest altitude being 250 kilometers and the highest 1000 kilometers. It is noted that at the point of crossing (i.e., at the line of vertices) the two transits do not have the same transit time. This is the result of the one parameter (injection longitude) variation. If constant transit time is desired at the line of vertices, then one of the other injection conditions will necessarily be varied.

In Figure 23 a comparison of the line of vertices location to that of the corresponding perpendicular impact is given for an injection longitude variation.

The lunar arrival longitudes are presented versus the transit time to perpendicular impact. The line of vertices is defined at the surface of the moon. In general, the line of vertices lies 180 degrees in lunar longitude from the perpendicular impact as noted in the figure. The relative location of both terminal conditions (i.e., perpendicular impact and line of vertices) is given for co-rotational and counter-rotational injection conditions at the earth.

5. The Free Return Transit

The free return transit is a special fly-by transit. It may be cislunar or circumlunar. In order to place the free return transit in perspective to the preceeding discussion, Figure 24 is presented. From Figure 6, it is noted that the perpendicular impact injection velocities associated with the 78 and 200 hour transit times are of the same magnitude. If this velocity is selected, two perpendicular impacts may be achieved by variations of injection longitude (Figure 7). In Figure 24 an injection longitude of 44 degrees gives a point on the locus of periselenum indicated by (1). Increasing injection longitude moves the terminal point of the transit along the locus of periselenum until a grazing impact is achieved at (2). Continued increase in the injection longitude changes the impact location until a perpendicular impact is achieved at point (3) (see insert, 78 hour perpendicular impact). Increasing the injection longitude to larger values brings the second grazing impact (4) (insert). Further increase in the injection longitude gives the first free return transit at the selected velocity (5) (transit time 83.2 hrs). Increasing the longitude to approximately 72 degrees brings the second free return transit for this velocity (6) (transit time 128 hrs). As injection longitude is increased beyond this point, the locus of periselenum returns to the surface of the moon and the second traverse of the lunar surface for this velocity begins at (7). The perpendicular impact associated with this traverse occurs at (8) having a transit time of 200 hours (note: perpendicular impact of first traverse was 78 hours transit time). The fourth grazing impact occurs at location (9). From this point, periselenum is again the termination criteria for the transits. At (10) the first cislunar free return transit occurs having a transit time of 208 hours. Continued increase in injection longitude moves the location of periselenum farther from the lunar surface as indicated at point (11). From point (1) to point (11) requires a variation in injection longitude of 76 degrees and results in a variation in transit time exceeding 150 hours. Three free return transits were noted for this injection velocity (10,863 meters per second) in the vicinity of the moon. Two of the free return transits were circumlunar and one cislunar. The associated transit times were 83, 128 and 208 hours. For the other locus of periselenum presented (v_E of 10,873 meters per second) there are two circumlunar free return transits of 72 and 163 hours at the x_R axis. For this velocity, there are no free return cislunar transits in the vicinity of the moon.

If the reference is consulted, it is noted that circumlunar free return transits of co-rotational injection require velocities between 10,880 meters per second and 10,859 meters per second. Cislunar transits require injection velocities between 10,880 meters per second and 10,847 meters per second or possibly to a lower value depending on allowable distance from the moon. In the case of circumlunar transits, when velocities lower than 10,859 meters per second are encountered one of two situations arise. Either the locus of periselenum is positioned such that it never crosses the x_R axis or the velocity is too low to achieve a circumlunar transit. In the latter case the periselenum is non-existent. For velocities in excess of 10,880 meters per second, the free return transit may exist but unfortunately below the lunar surface.

REFERENCE

MTP-AERO-63-14, "Lunar Flight Study Series: Volume 5, Trajectories in the Earth - Moon Space with Symmetrical Free Return Properties," by Arthur J. Schwaniger, February 8, 1963.

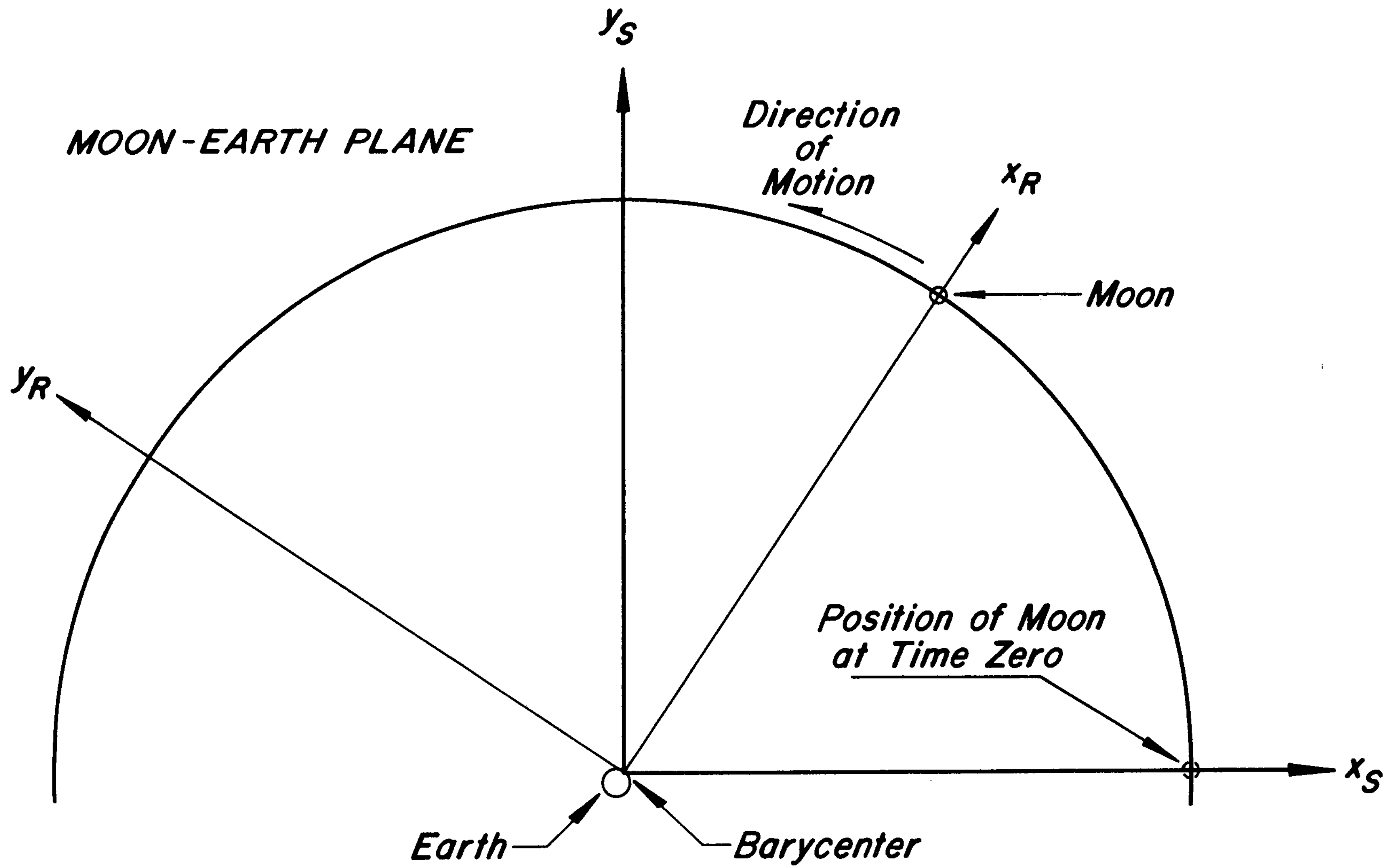


FIG. I. SPACE FIXED (x_S, y_S) AND ROTATING (x_R, y_R) COORDINATE SYSTEMS

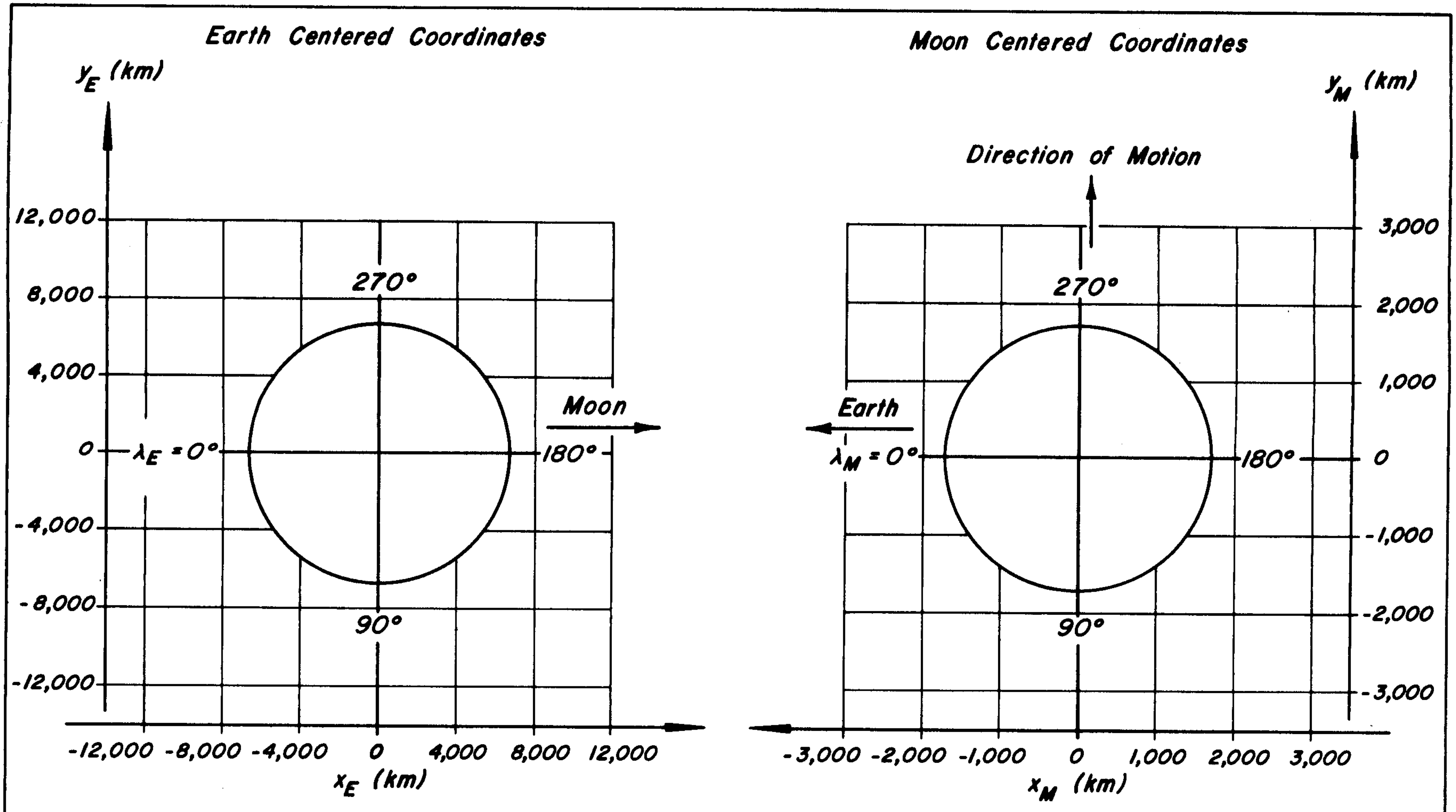


FIG. 2. EARTH CENTERED (x_E, y_E) AND MOON CENTERED (x_M, y_M) COORDINATE SYSTEMS (MOON-EARTH PLANE)

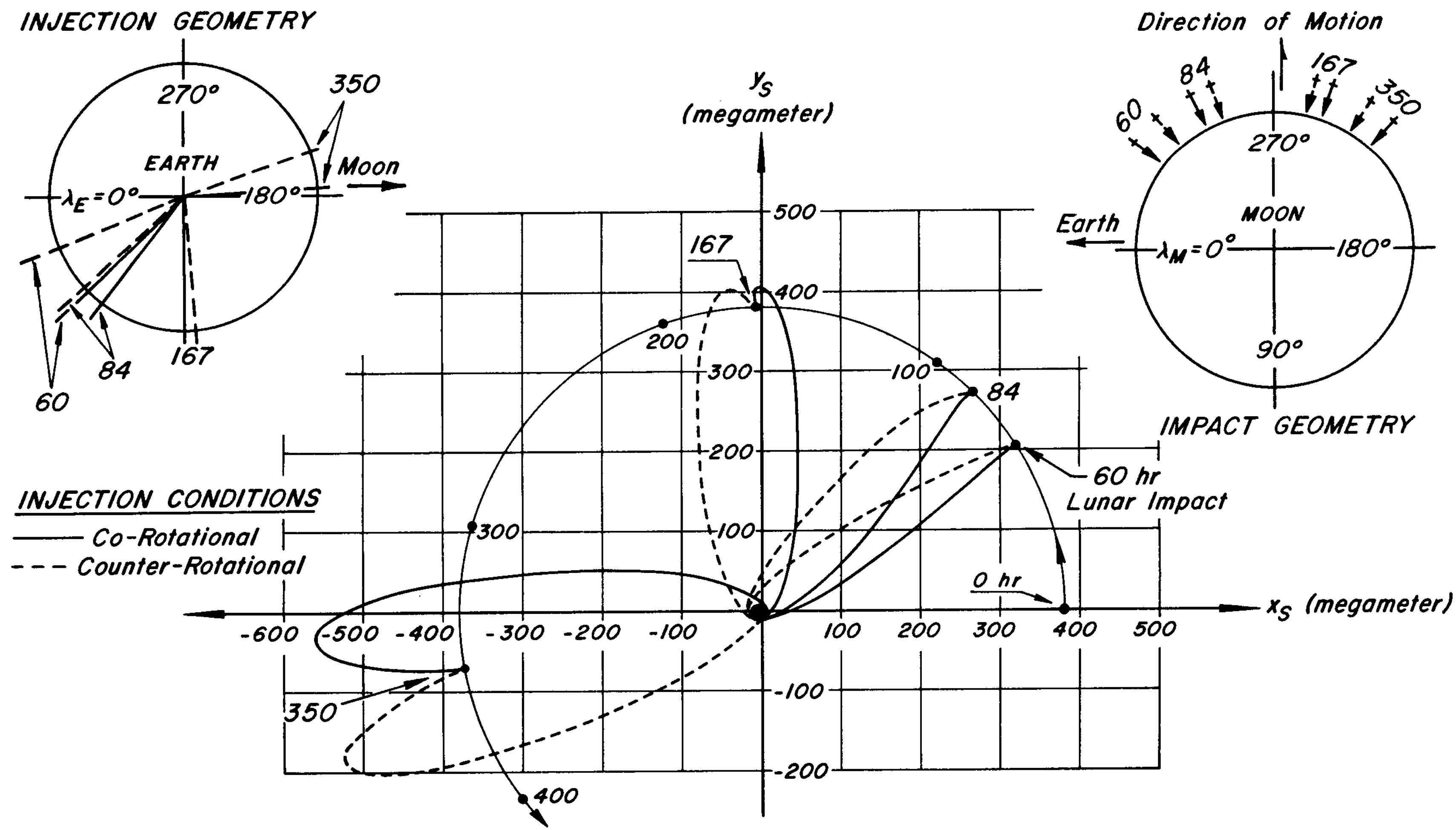


FIG. 3. COPLANAR EARTH-MOON TRANSITS FOR PERPENDICULAR LUNAR IMPACTS PRESENTED IN A SPACE-FIXED REFERENCE SYSTEM MOON-EARTH PLANE

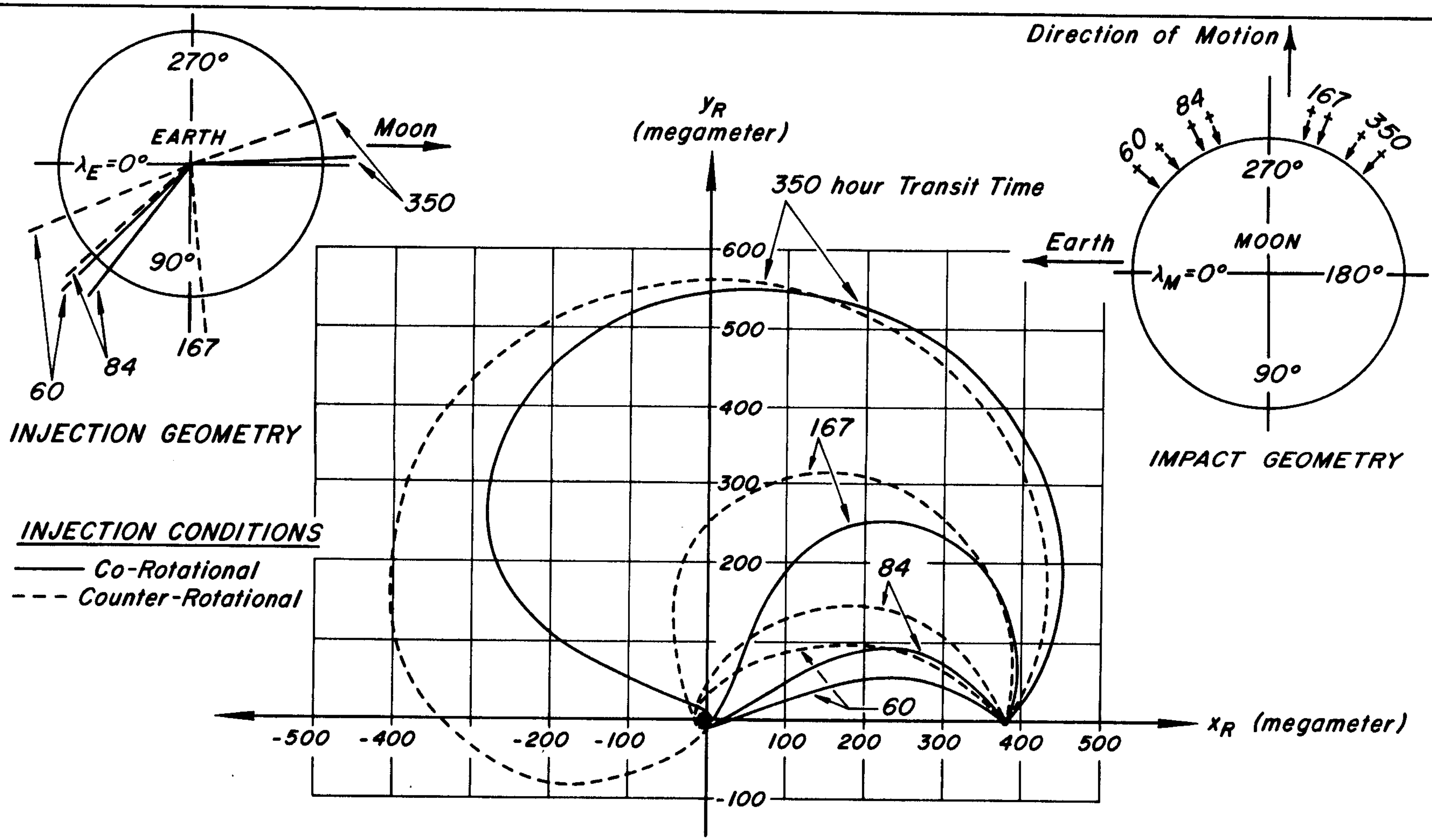


FIG. 4. COPLANAR EARTH-MOON TRANSITS FOR PERPENDICULAR LUNAR IMPACTS PRESENTED IN A ROTATING REFERENCE SYSTEM MOON-EARTH PLANE

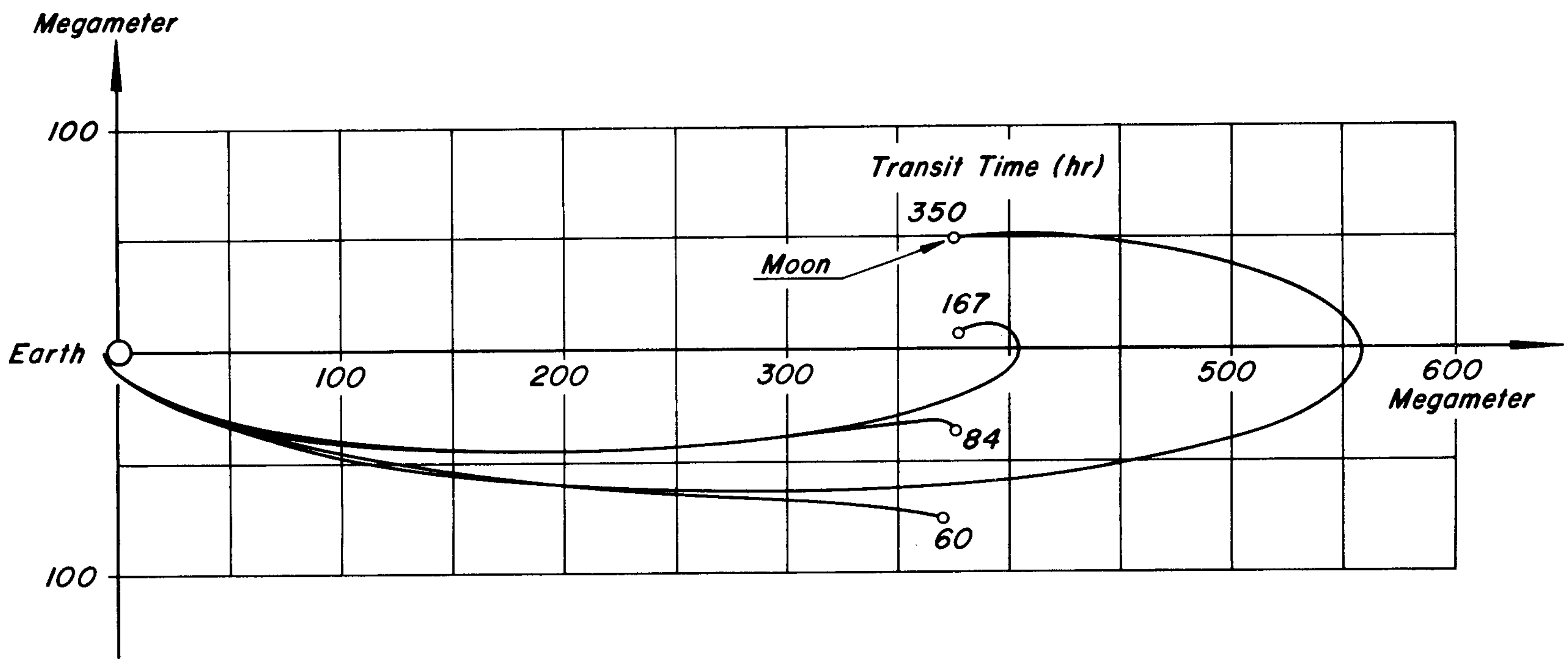
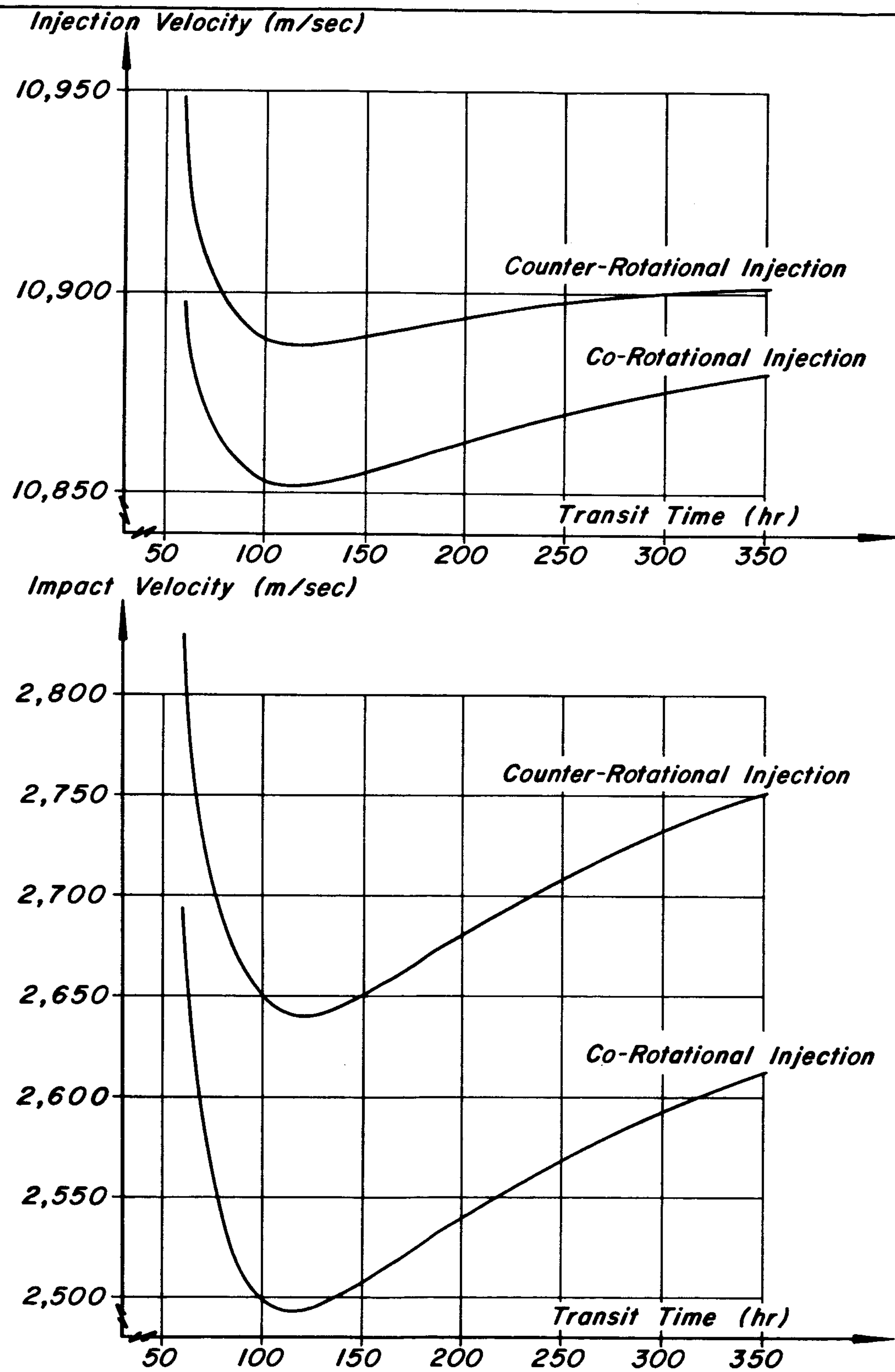


FIG. 5. COPLANAR EARTH-MOON TRANSITS FOR PERPENDICULAR LUNAR IMPACTS IN AN INERTIAL COORDINATE SYSTEM WITH COINCIDENT LINES OF APSIDES

MOON-EARTH PLANE

CO-ROTATIONAL INJECTION



F'G. 6. TERMINAL VELOCITIES FOR EARTH-MOON TRANSITS WITH PERPENDICULAR IMPACTS VERSUS TRANSIT TIME IN MOON-EARTH PLANE

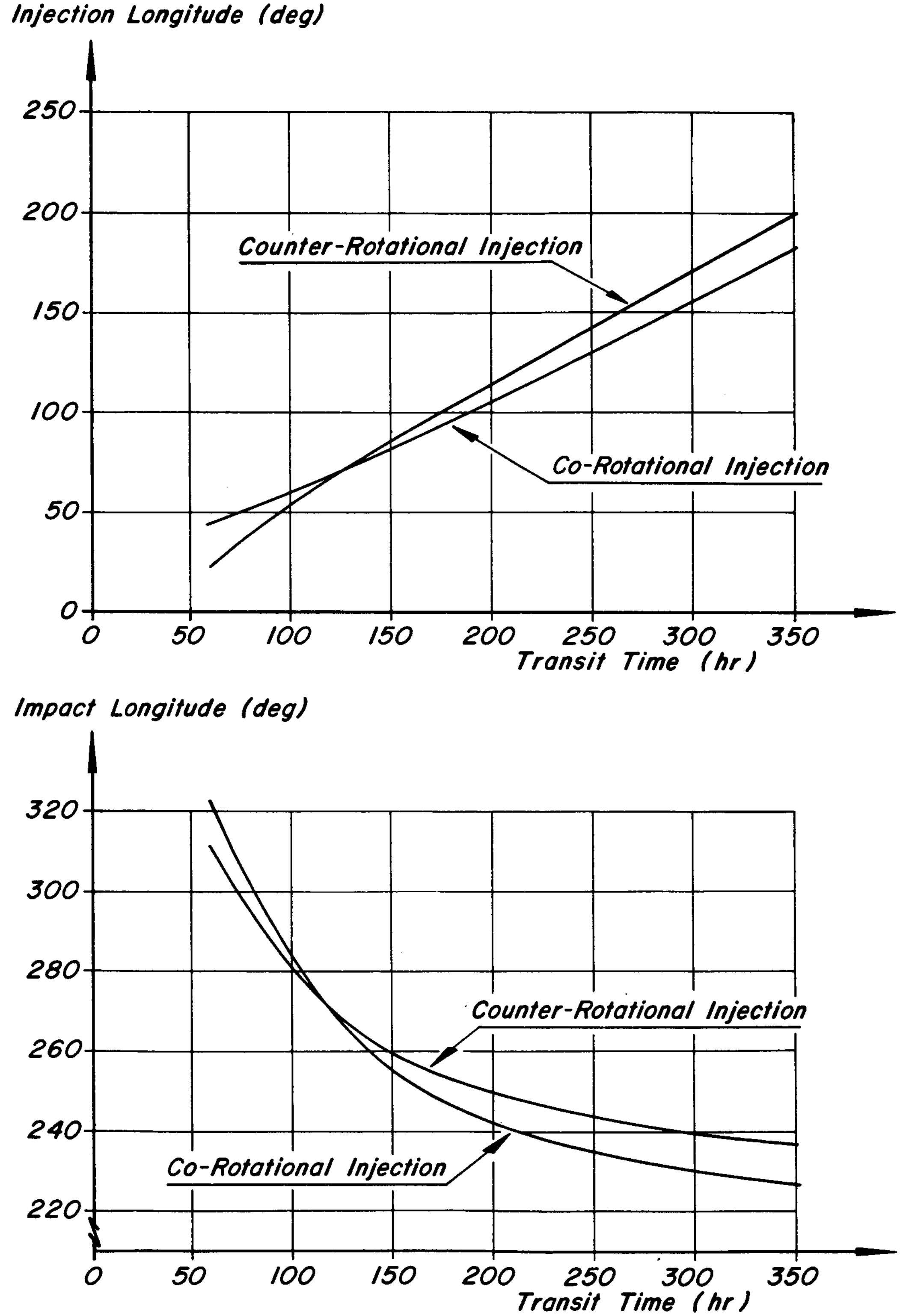


FIG. 7. TERMINAL LONGITUDES FOR EARTH-MOON TRANSITS WITH PERPENDICULAR IMPACTS VERSUS TRANSIT TIME IN MOON-EARTH PLANE

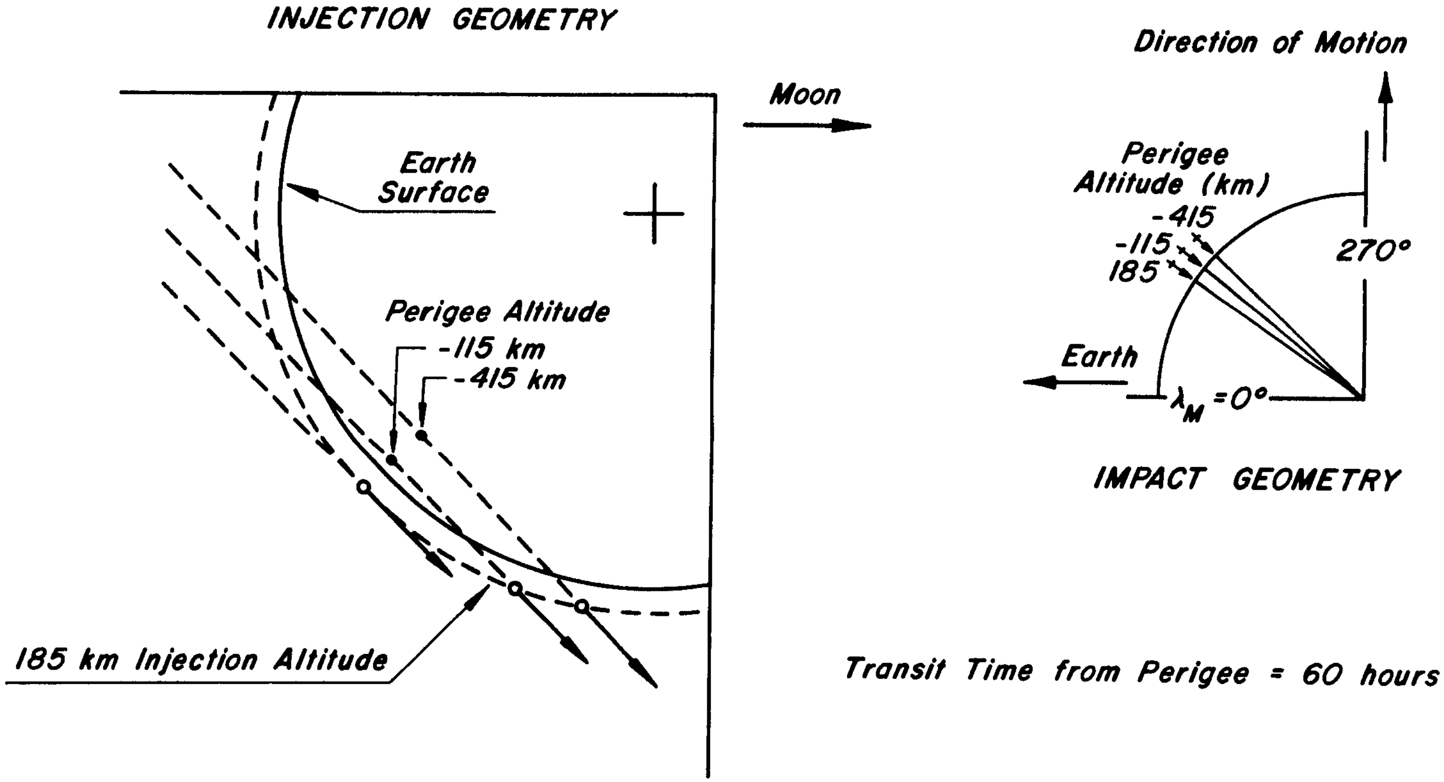


FIG. 8. INJECTION AND IMPACT GEOMETRY FOR EQUAL TRANSIT TIME PERPENDICULAR IMPACT TRANSITS FROM VARIED PERIGEE ALTITUDES

MOON-EARTH PLANE CO-ROTATIONAL INJECTION

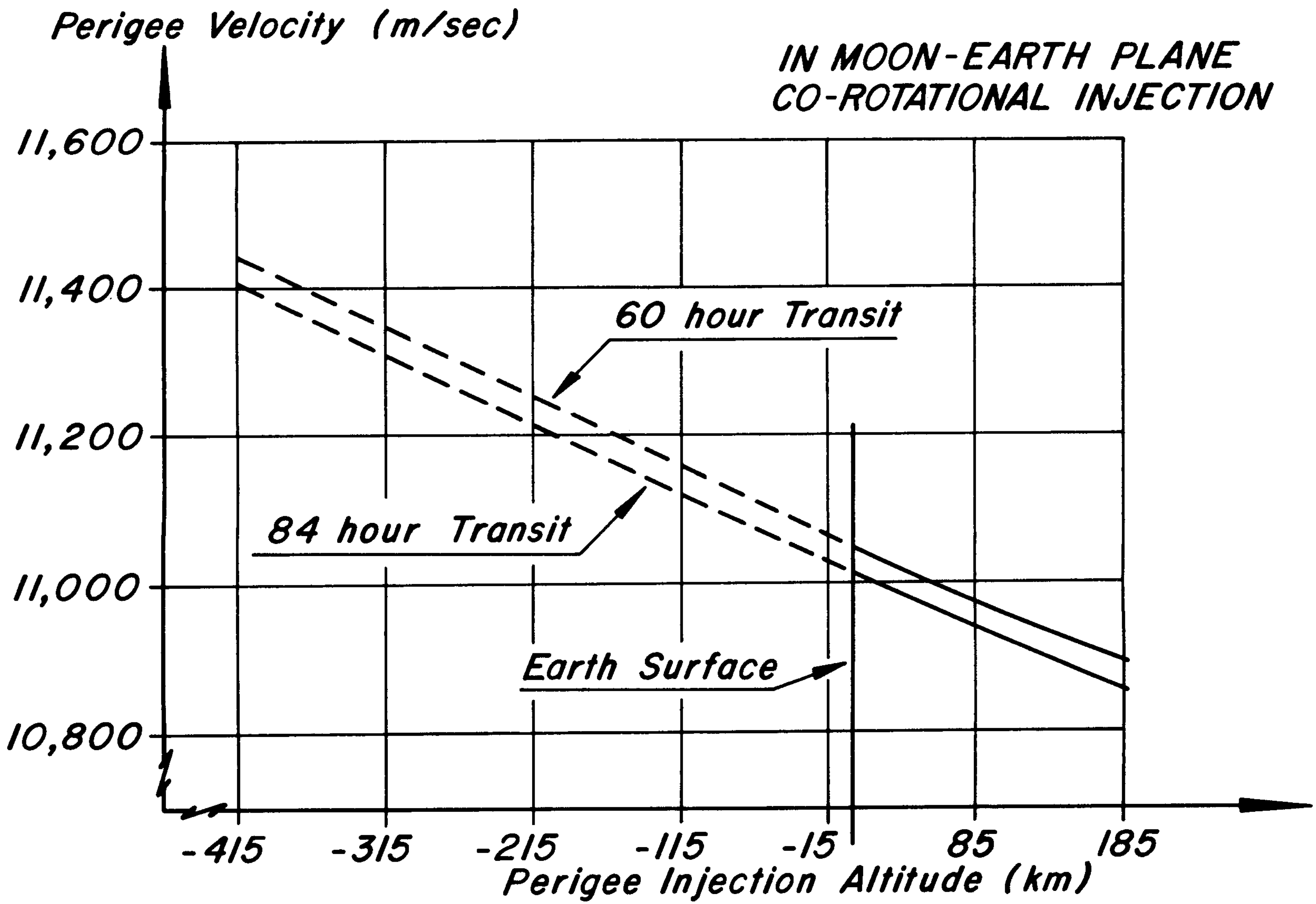


FIG. 9. PERIGEE VELOCITIES
FOR EQUAL TRANSIT TIME PERPENDICULAR IMPACT TRANSITS
FROM VARIED PERIGEE ALTITUDES

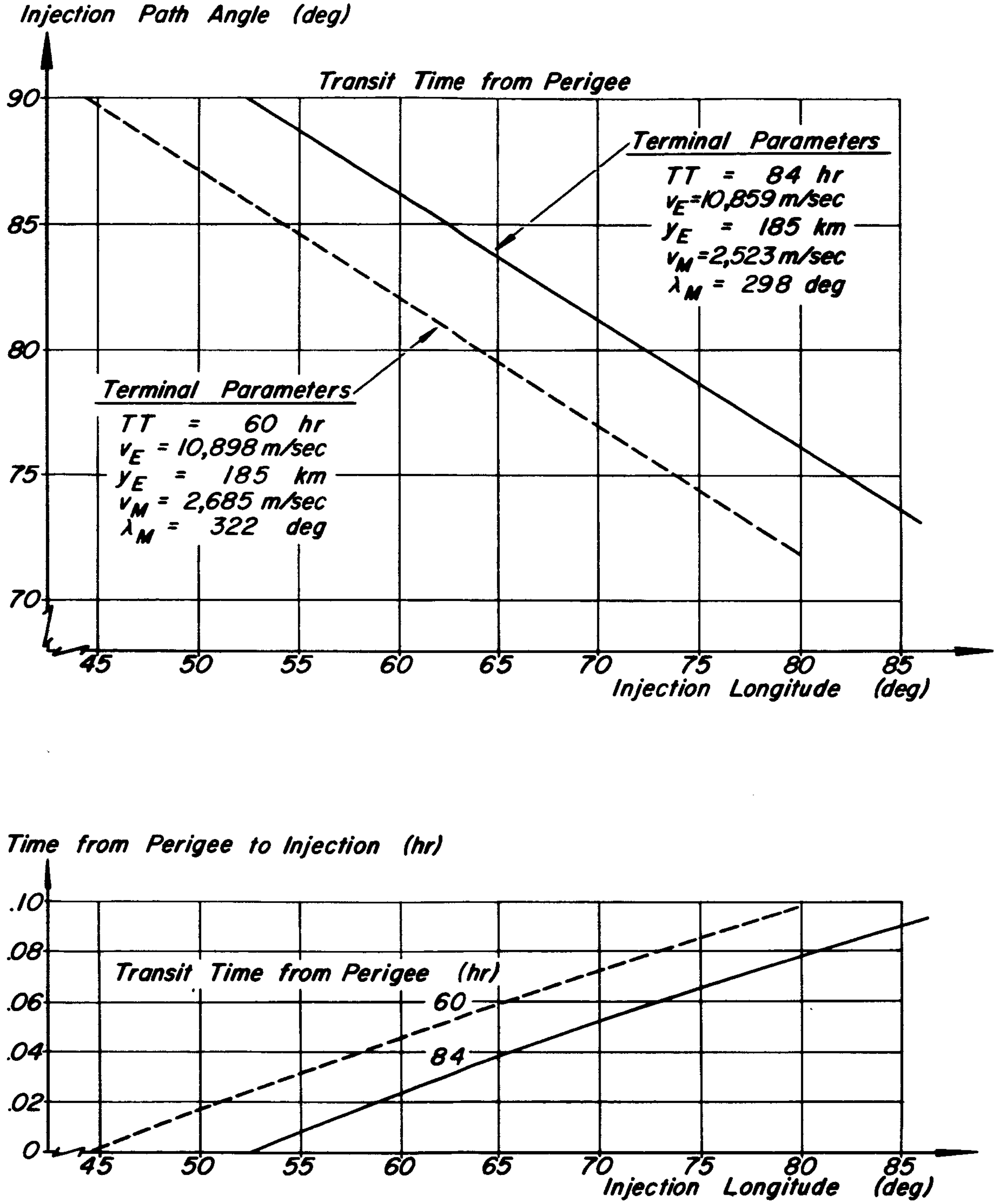


FIG. 10. CONSTANT INJECTION ALTITUDE CONDITIONS FOR EQUAL TRANSIT TIME PERPENDICULAR IMPACT TRANSITS FROM VARIED PERIGEE ALTITUDES

IN MOON-EARTH PLANE CO-ROTATIONAL INJECTION

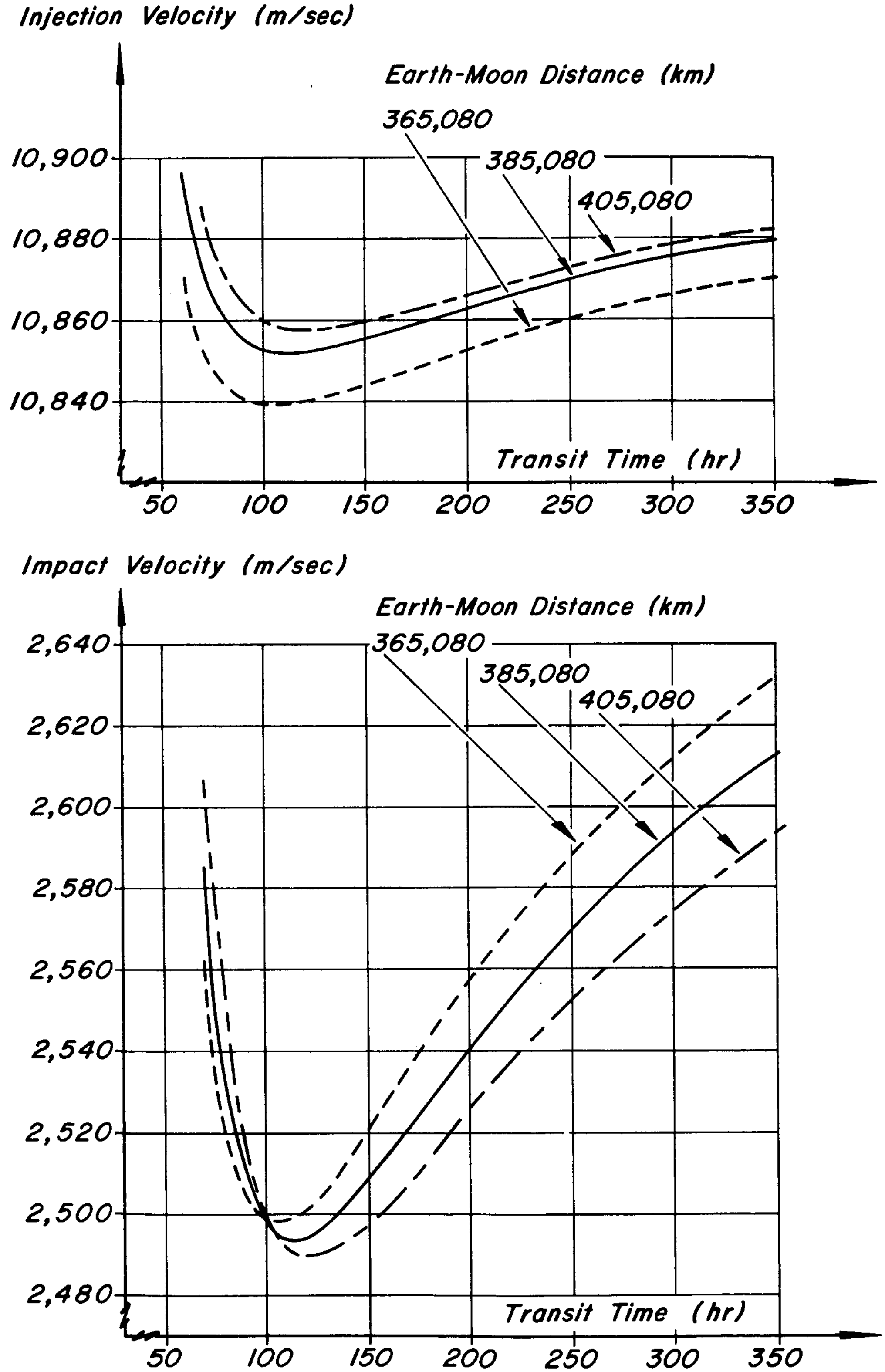


FIG. 11. TERMINAL VELOCITIES FOR EARTH-MOON TRANSITS WITH PERPENDICULAR IMPACTS AT VARIED EARTH-MOON DISTANCES IN MOON-EARTH PLANE CO-ROTATIONAL INJECTION

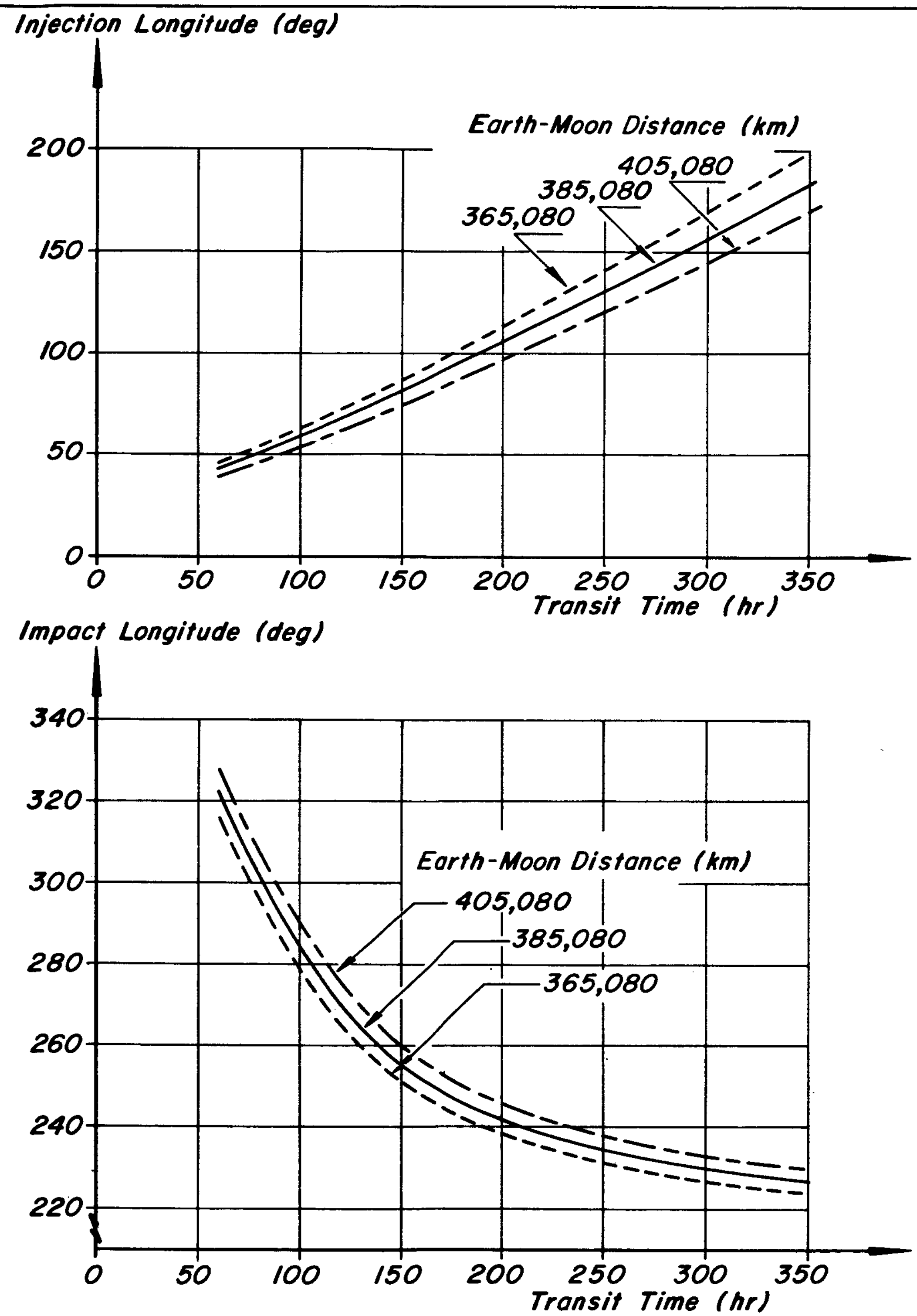


FIG. 12. TERMINAL LONGITUDES FOR EARTH-MOON TRANSITS WITH PERPENDICULAR IMPACTS AT VARIED EARTH-MOON DISTANCES

IN MOON-EARTH PLANE CO-ROTATIONAL INJECTION

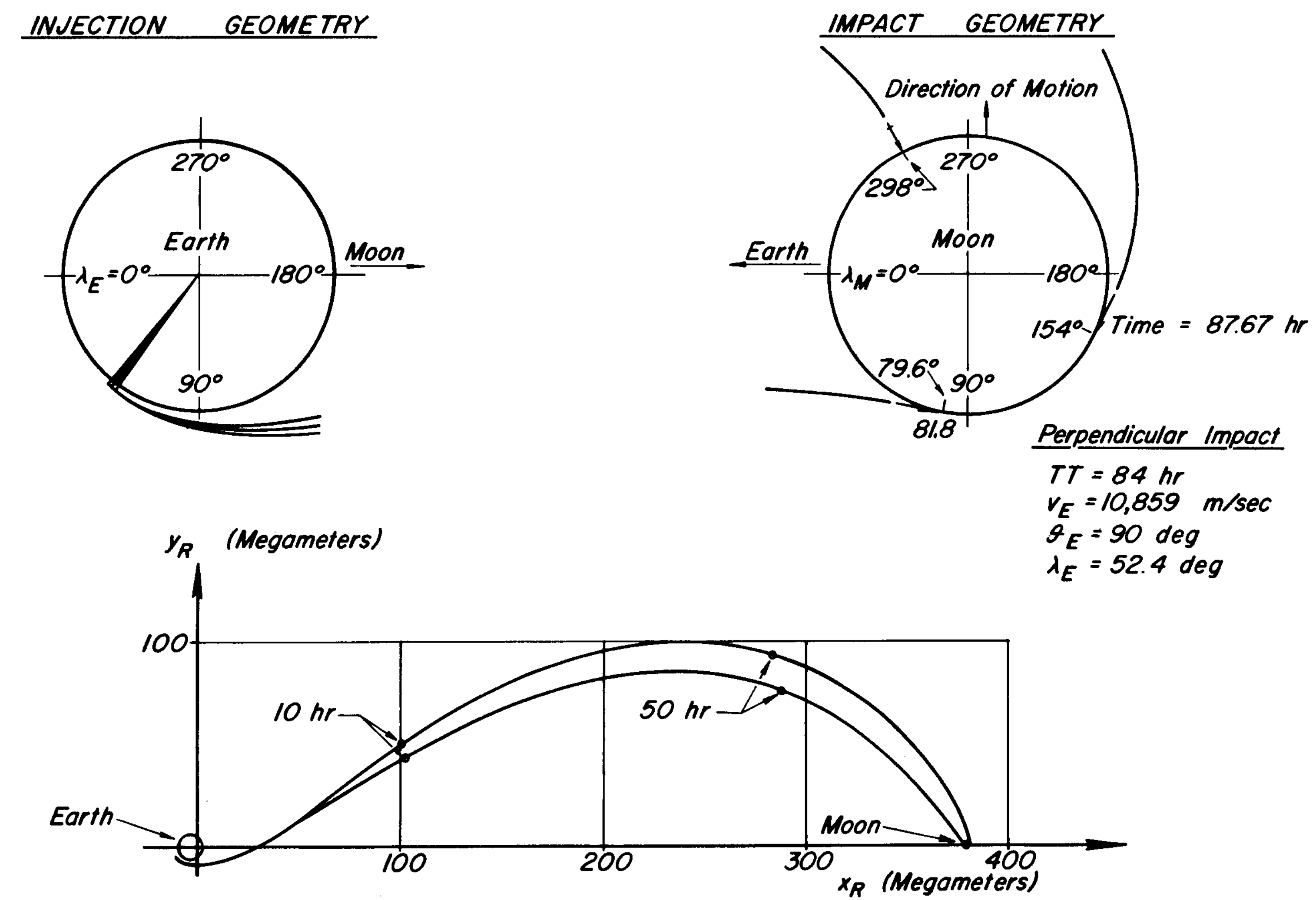


FIG. 13. EARTH-MOON TRANSIT GEOMETRY FOR NON-PERPENDICULAR IMPACTS
MOON-EARTH PLANE CO-ROTATIONAL INJECTION

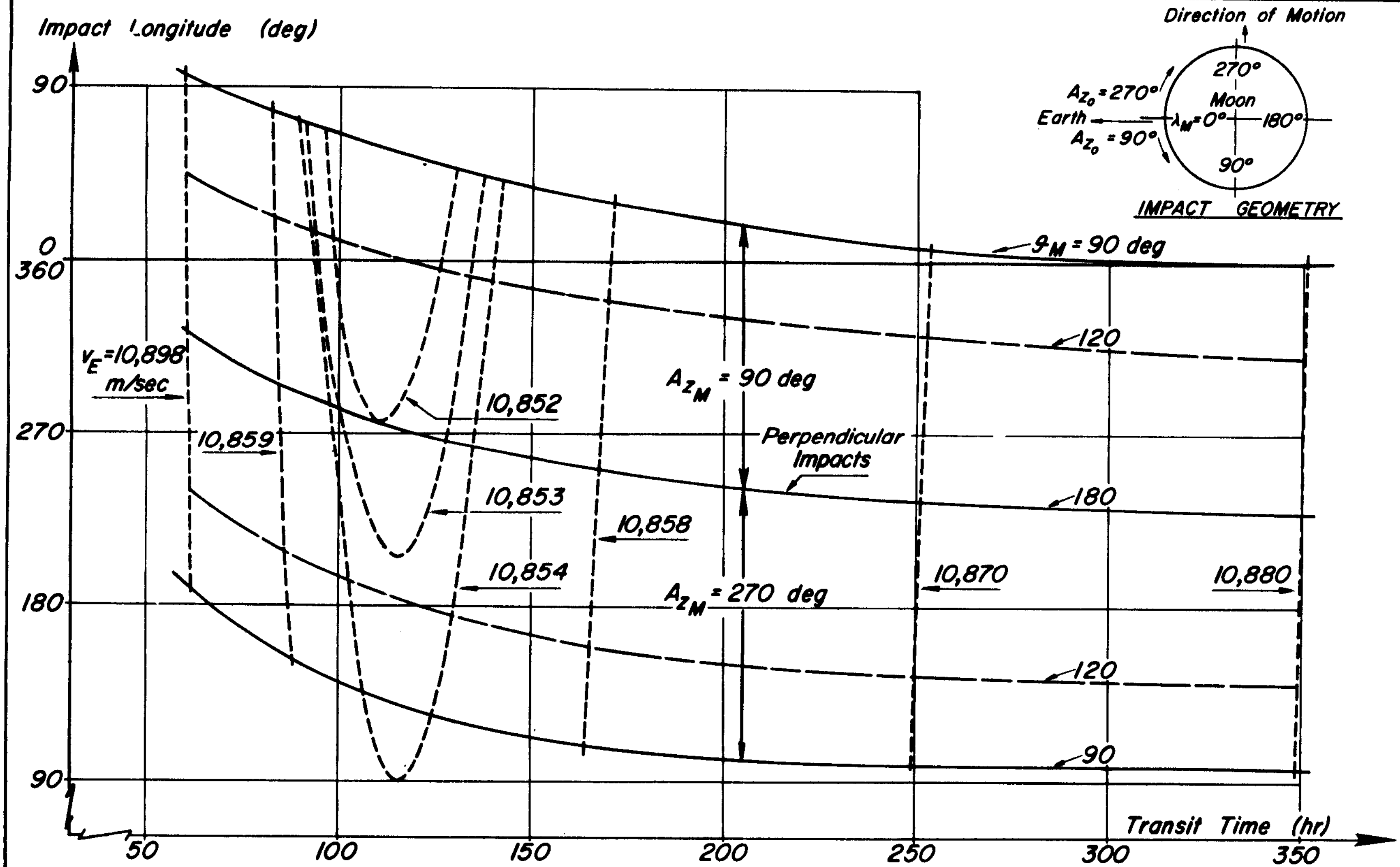


FIG. 14. IMPACT LONGITUDE VERSUS TRANSIT TIME FOR INJECTION LONGITUDE VARIATIONS IN MOON-EARTH PLANE CO-ROTATIONAL INJECTION

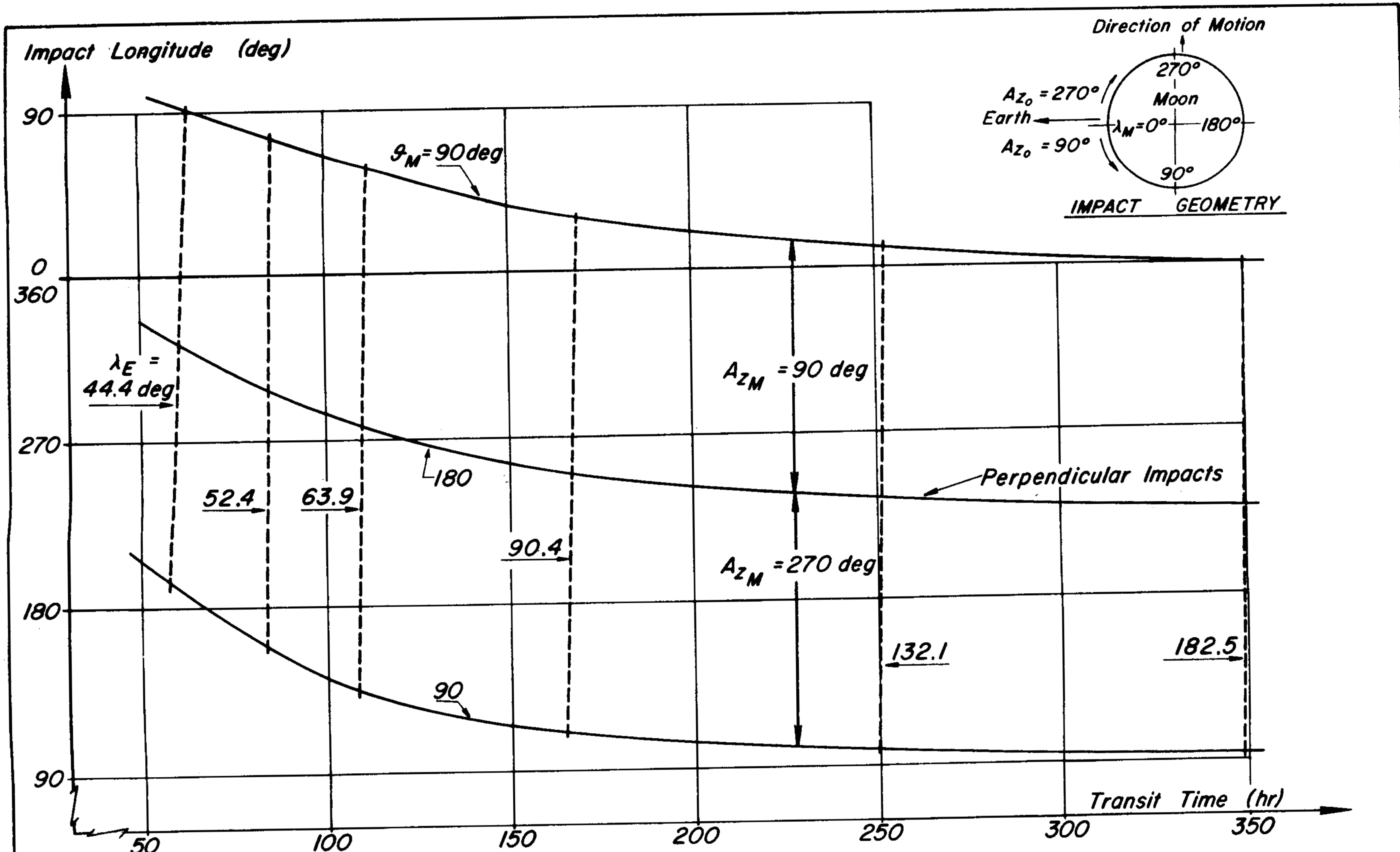


FIG. 15. IMPACT LONGITUDE VERSUS TRANSIT TIME FOR INJECTION VELOCITY VARIATIONS IN MOON-EARTH PLANE CO-ROTATIONAL INJECTION

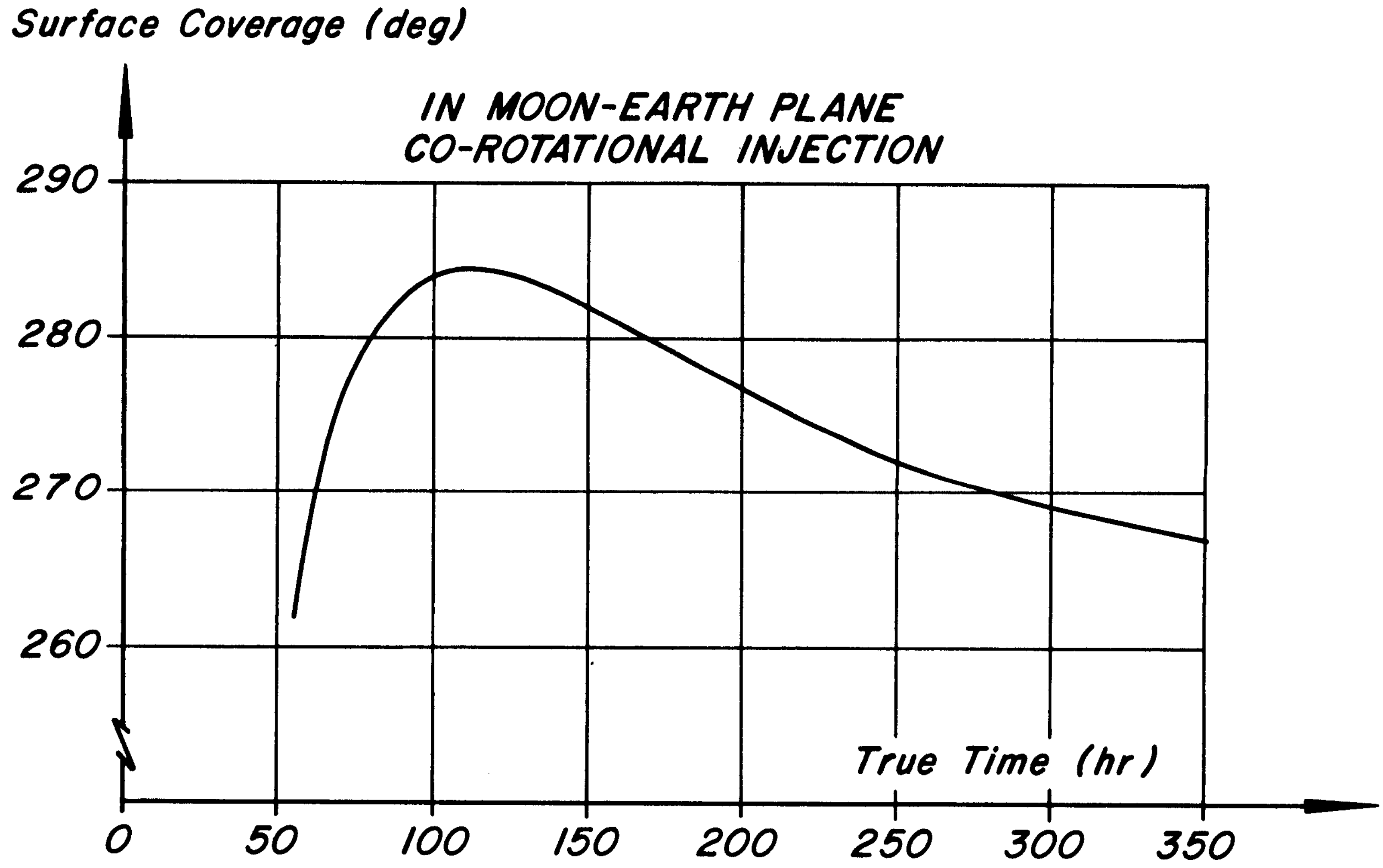


FIG. 16. LUNAR SURFACE COVERAGE VERSUS TRANSIT TIME FOR INJECTION VELOCITY OR LONGITUDE VARIATIONS

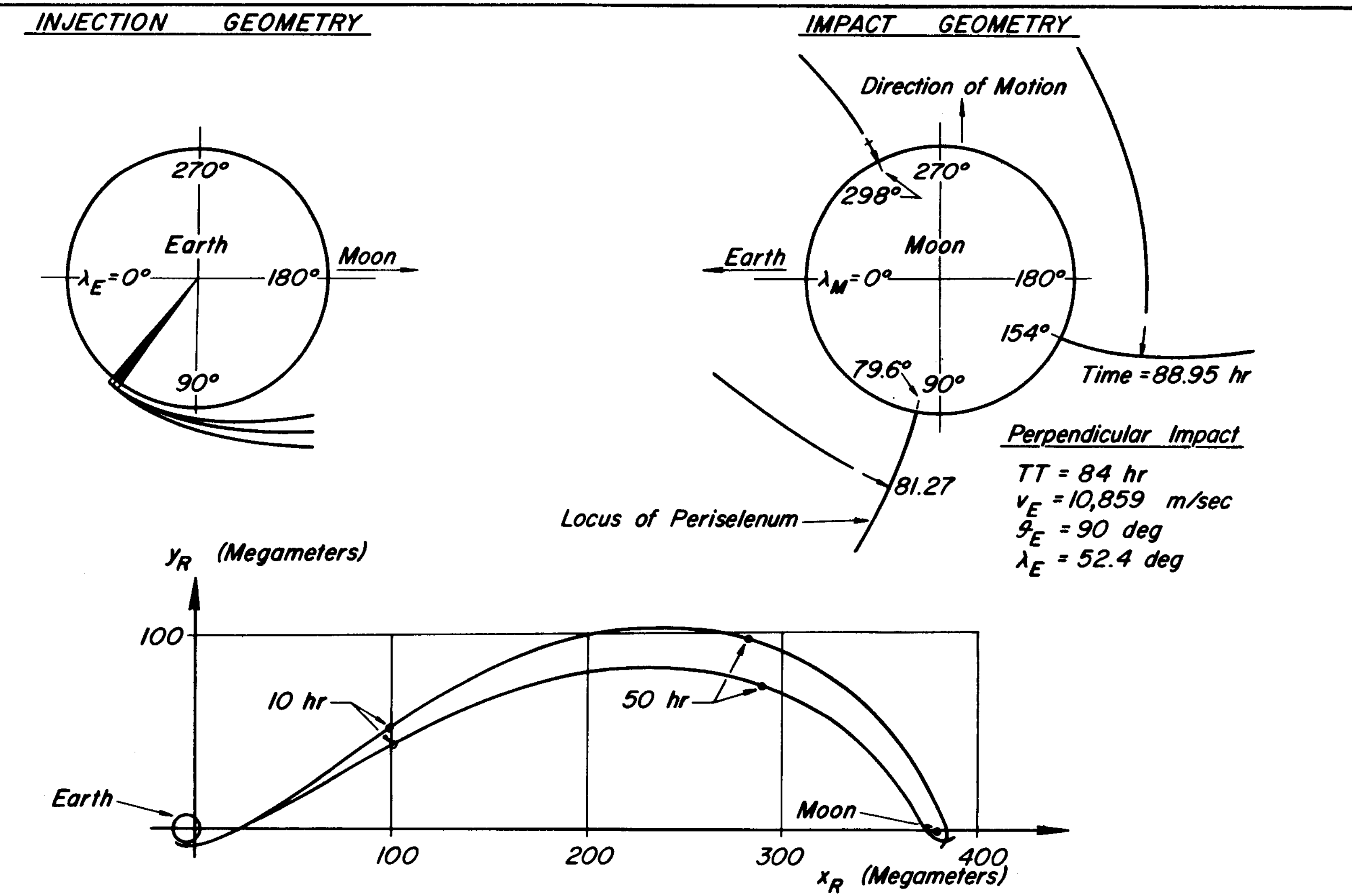


FIG. 17. EARTH-MOON TRANSIT GEOMETRY FOR LUNAR FLY-BY TRANSITS
MOON-EARTH PLANE CO-ROTATIONAL INJECTION

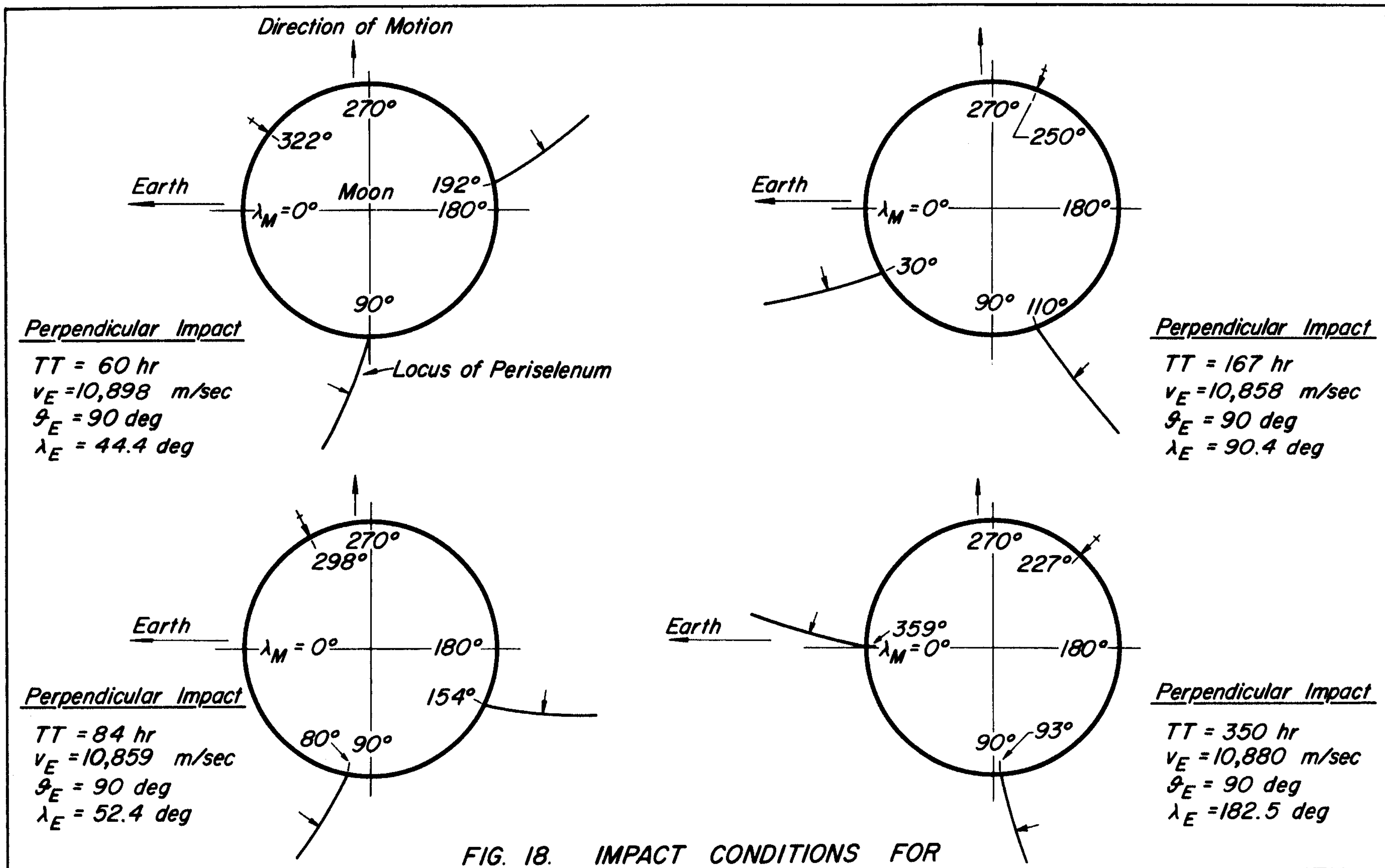


FIG. 18. IMPACT CONDITIONS FOR PERPENDICULAR IMPACTS AND CORRESPONDING PERISELENA FOR EQUAL INJECTION VELOCITY
MOON-EARTH PLANE CO-ROTATIONAL INJECTION INJECTION LONGITUDE VARIATIONS

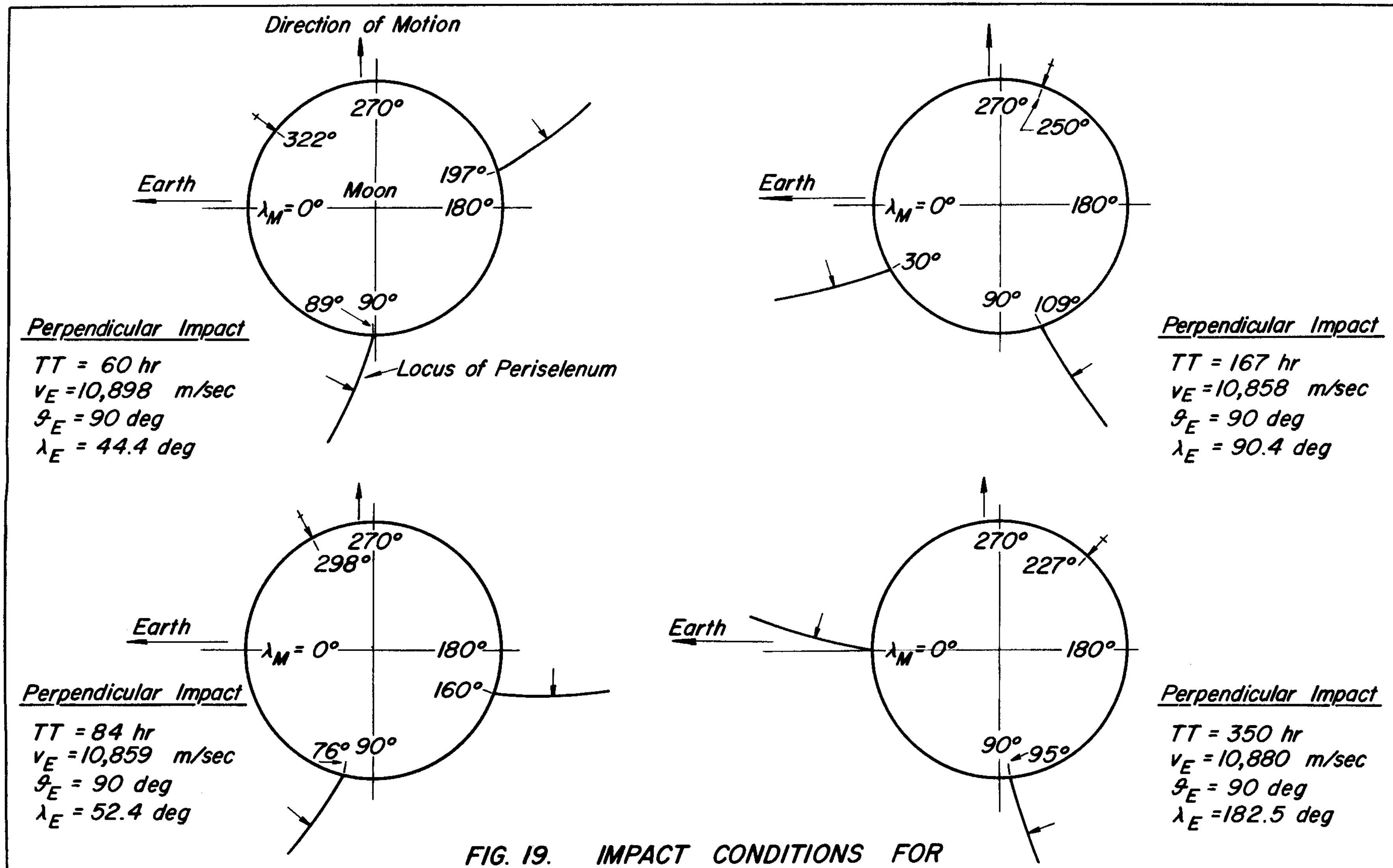


FIG. 19. IMPACT CONDITIONS FOR PERPENDICULAR IMPACTS AND CORRESPONDING PERISELENA FOR EQUAL INJECTION LONGITUDES

MOON-EARTH PLANE CO-ROTATIONAL INJECTION INJECTION VELOCITY VARIATIONS

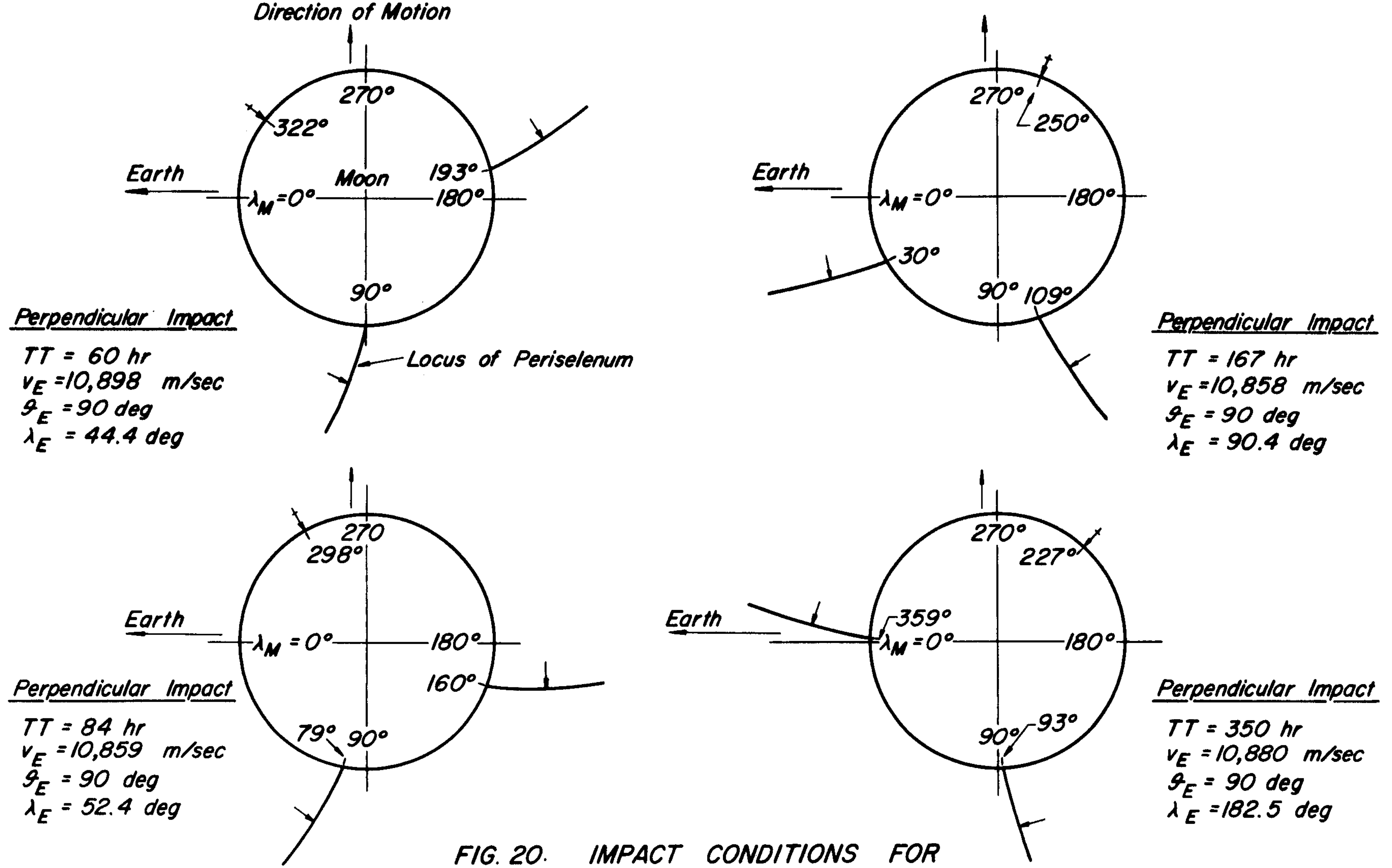


FIG. 20. IMPACT CONDITIONS FOR PERPENDICULAR IMPACTS AND CORRESPONDING PERISELENA FOR CONSTANT TRANSIT TIMES MOON-EARTH PLANE CO-ROTATIONAL INJECTION INJECTION VELOCITY AND LONGITUDE VARIATIONS

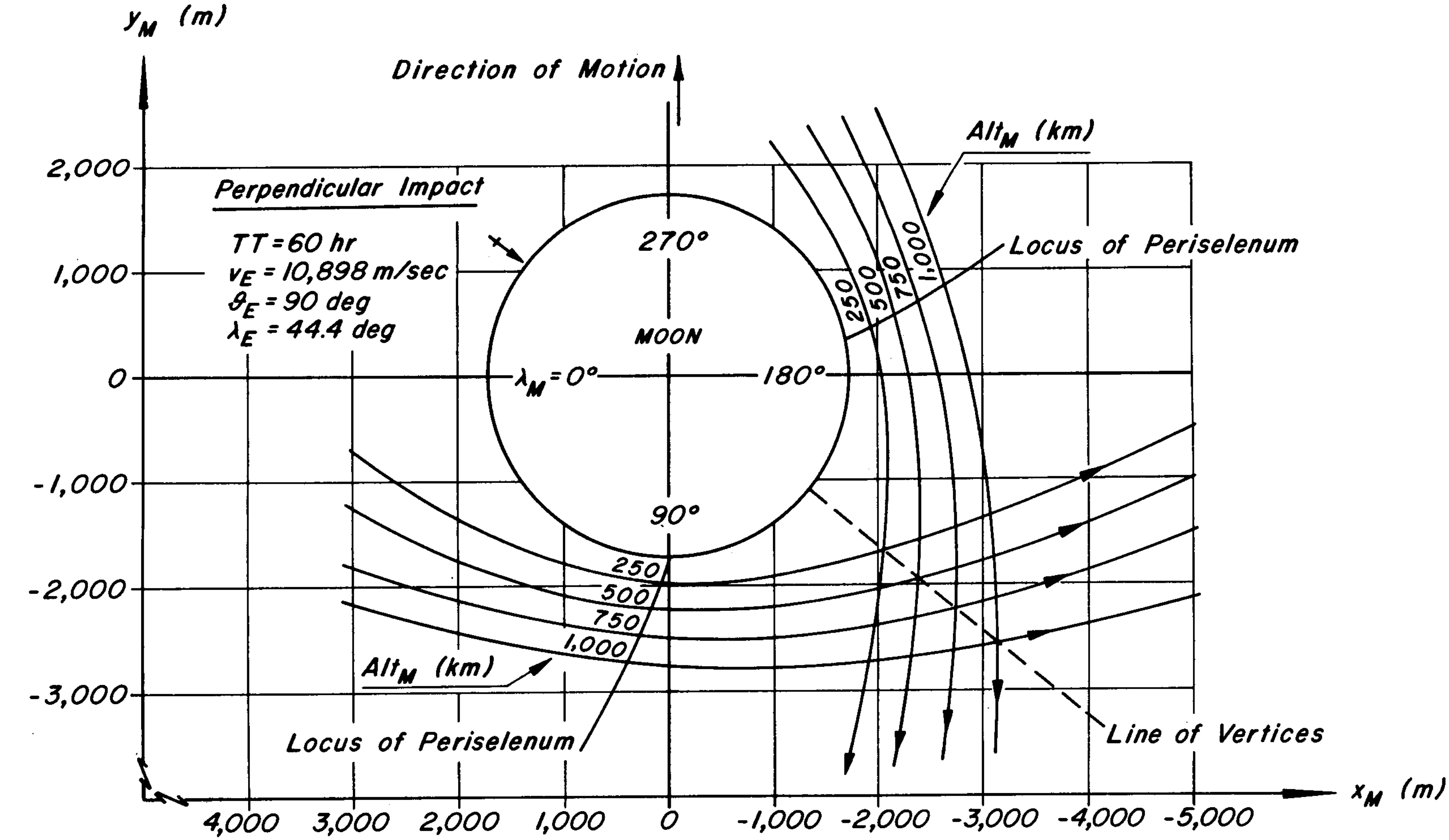


FIG. 21. THE LINE OF VERTICES AS INDICATED BY TRANSITS OF CONSTANT INJECTION VELOCITY AND CONSTANT LUNAR ALTITUDE

MOON-EARTH PLANE CO-ROTATIONAL INJECTION INJECTION LONGITUDE VARIATIONS

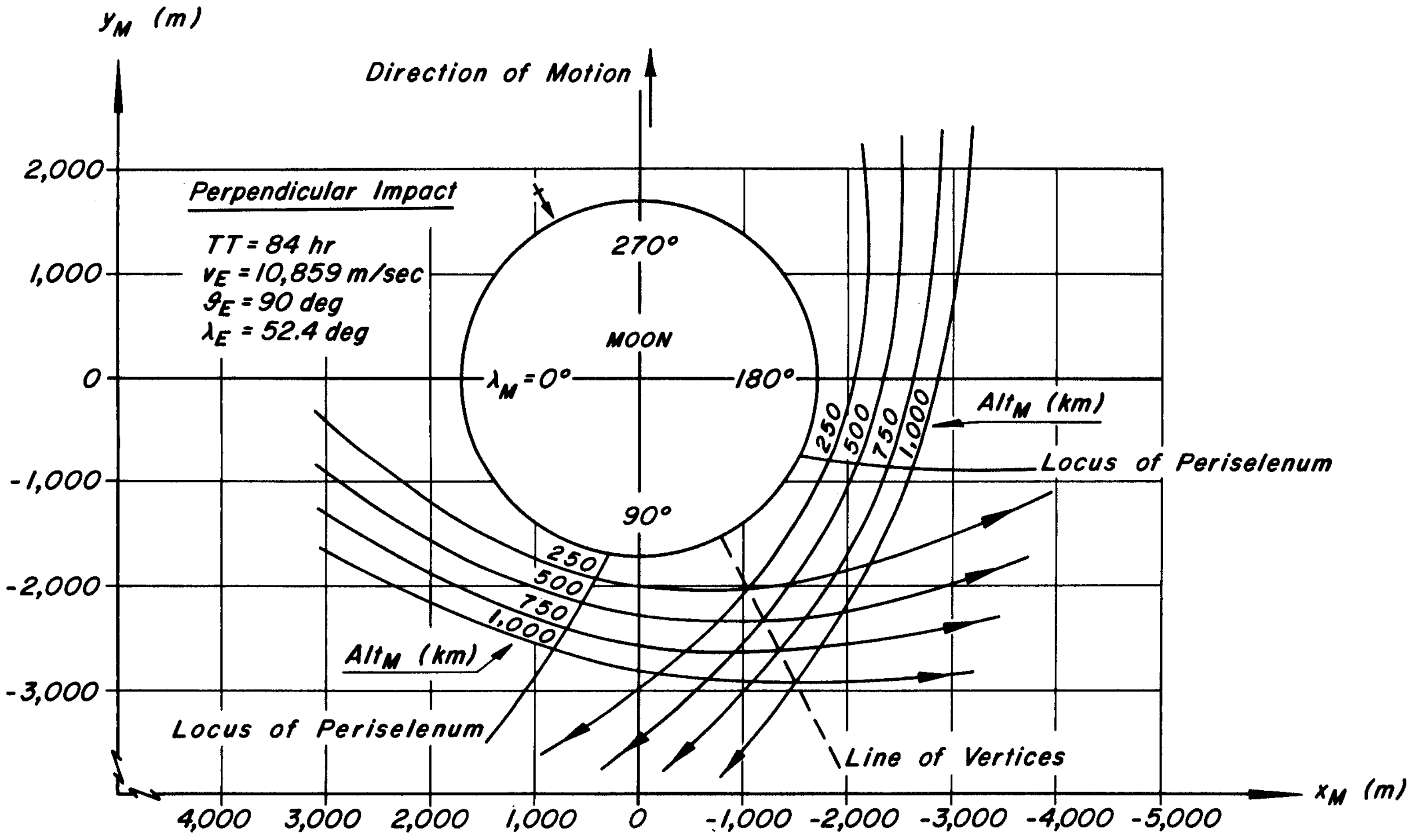


FIG. 22. THE LINE OF VERTICES AS INDICATED BY TRANSITS OF CONSTANT INJECTION VELOCITY AND CONSTANT LUNAR ALTITUDE
MOON-EARTH PLANE CO-ROTATIONAL INJECTION INJECTION LONGITUDE VARIATIONS

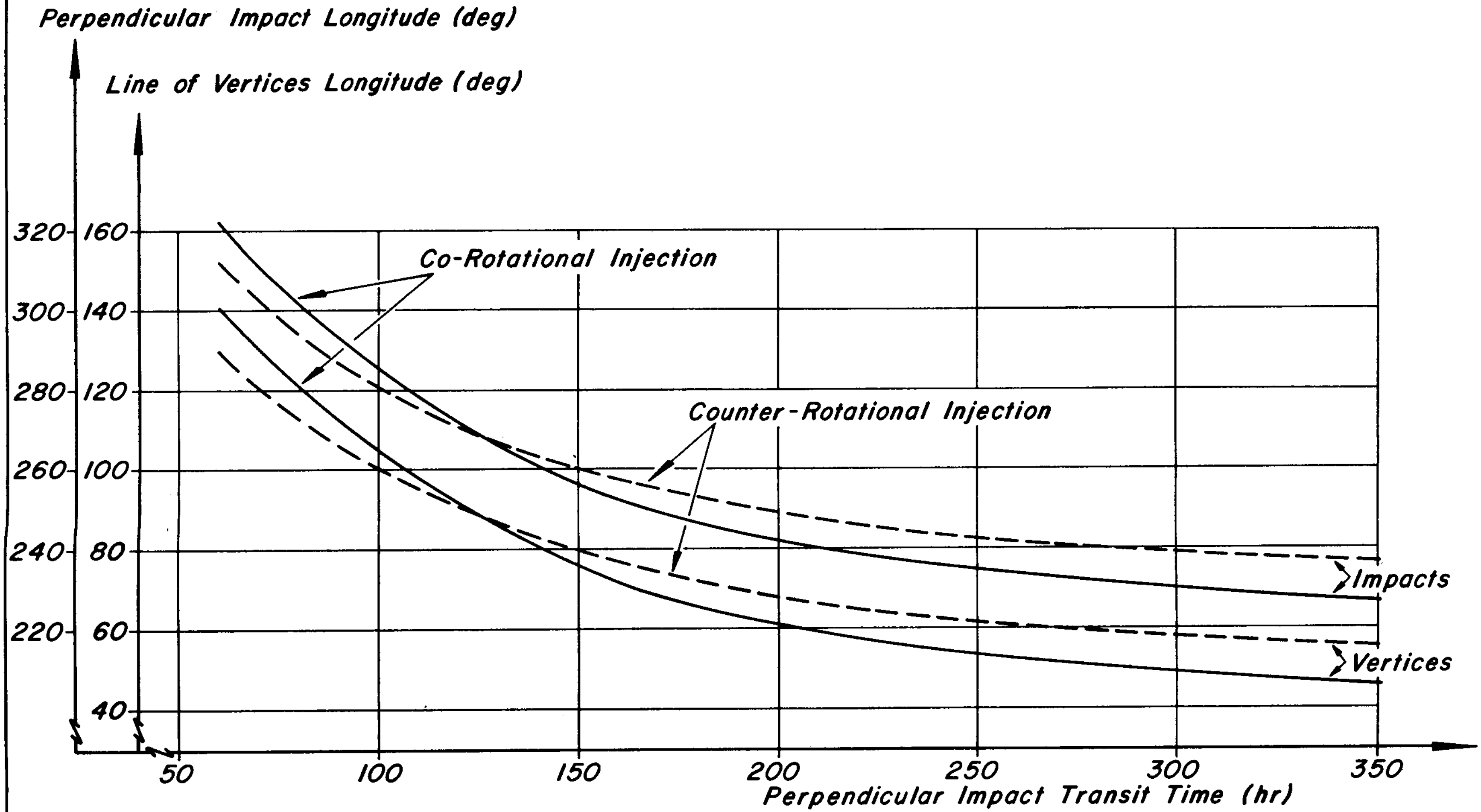


FIG. 23. LUNAR LONGITUDE OF PERPENDICULAR IMPACT AND LINE OF VERTICES VERSUS TRANSIT TIME (IN MOON-EARTH PLANE)

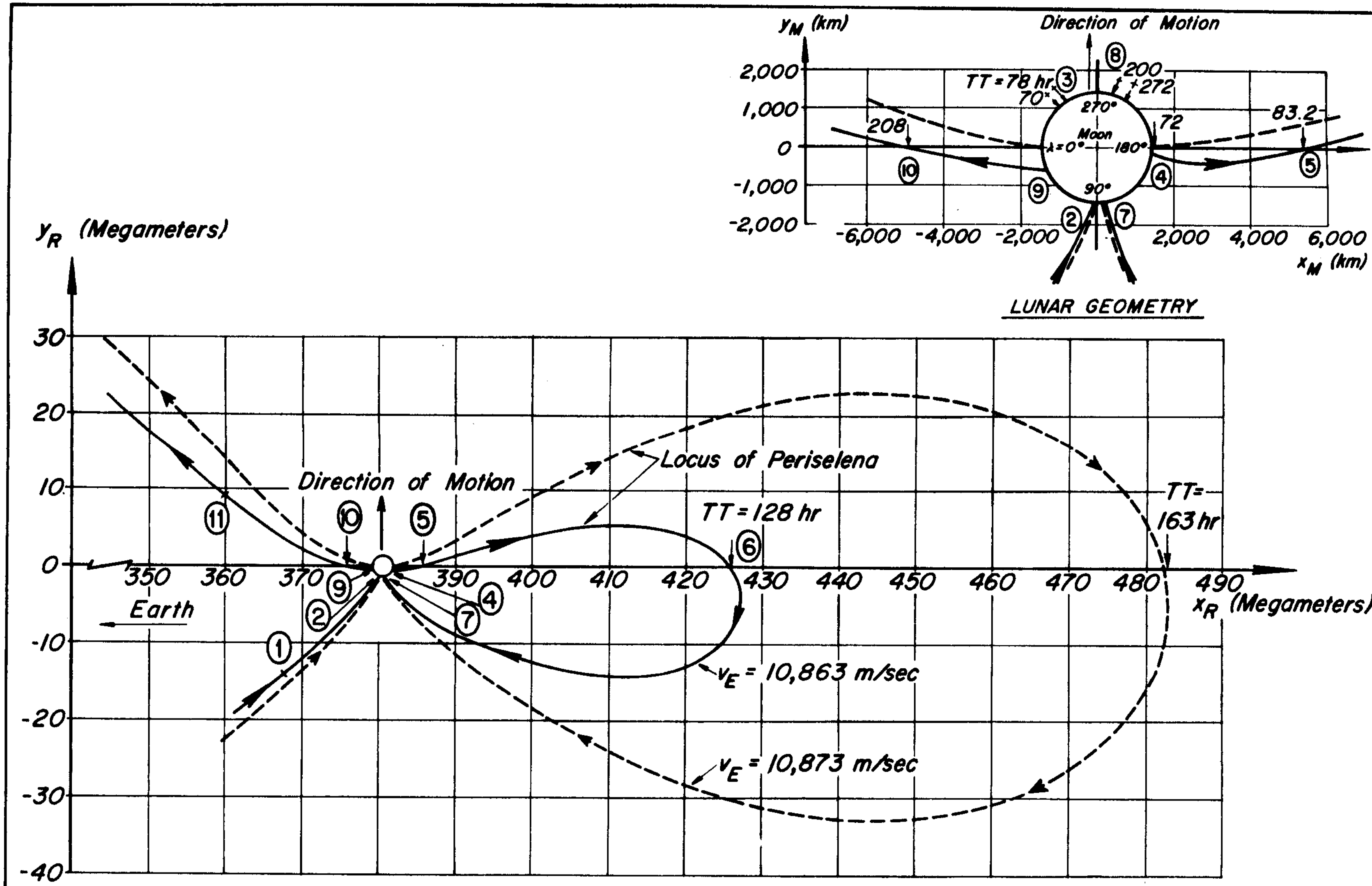


FIG. 24. GEOMETRICAL RELATIONSHIP OF FREE RETURN TRANSITS TO LOCUS OF PERISELENUM
MOON-EARTH PLANE CO-ROTATIONAL INJECTION INJECTION ALTITUDE = 185 KM

APPROVAL

LUNAR FLIGHT STUDY SERIES: VOLUME 6

A STUDY OF GEOMETRICAL AND TERMINAL CHARACTERISTICS OF EARTH-MOON TRANSITS EMBEDDED IN THE EARTH-MOON PLANE

By B. J. Lisle

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be Unclassified.

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